Endogenously Informed Boards and Board Power

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Abstract

This paper develops a theory of board power when the board of directors collects private information about the CEOs ability to make value-increasing investments. When CEO ability is unknown ex-ante, we show that board power can be helpful in inducing information production by the board and in eliminating over-investment by managers with low ability. However, we find that board power also comes at a cost of rejecting good investments by highly talented managers when the board is uninformed and hence has to rely on noisy public information. The paper thus highlights the importance of the interaction between board power and the public and private information environment faced by the board. Modeling explicitly the power of the board to reject major investment decisions we derive several novel implications on how board power impact managerial turnover, managerial investment, and overall firm value. For example, we find conditions under which more powerful boards will be associated with a lower sensitivity of managerial turnover to negative market signals as well as in boards that discourage efficient investments. The paper highlights some of the costs associated with awarding the board of directors with too much power and hence identifies the conditions under which increases in board power enhance shareholder value.

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1. Introduction

An essential component of a good corporate governance system is to have strong boards that are not beholden to the CEO. Stronger boards benefit shareholders because they have a greater ability to mitigate and limit various forms of agency costs. This logic, for example, led to the legislation of increasing the number of independent directors who serve on the board in order to limit the control of the CEO over the company’s board of directors.

While there are clear benefits to giving the board of directors more power to either approve or reject major investment decisions, we argue that there may also be costs associated with greater board control. Our goal in this paper is to construct a formal model of board power and to analyze the benefits and costs of allocating more power to the board and less power to the CEO. The equilibrium we focus on highlights the important interaction between board power and the (endogenous) private information collected by the board as well as the (exogenous) public information available to all investors. The model allows us to identify several new theoretical implications on whether or not stronger boards lead to higher firm value, on whether stronger boards lead to a reduction of innovative investments, and on how board power relates to the decision to replace the CEO.

We analyze these issues in a setting where the manager’s talent level is uncertain and is, initially, not observed by either the board of directors or by the market. The manager is faced with a decision of whether or not to make a significant investment which requires board approval. The agency problem stems from the fact that all managers, regardless of their type, receive private benefits from investing. However, the investment benefits shareholders only when it is managed by a talented CEO. Low talent managers should not invest. Thus, our model considers a situation where highly talented managers do not create agency costs while low talent managers generate an over-investment problem.

The role of the board then is to try and learn about the manager’s skill level and use its power to approve the investment if it is in the interest of shareholders and reject the investment if it is detrimental to shareholder value. In addition to the board’s private
information about the talent level of the CEO we consider the possibility that the market too produces a noisy signal about the CEO’s talent.

The resulting equilibrium allows us to analyze the costs and benefits of giving the board power over decisions. The key tradeoff we identify is that giving the board power is beneficial to shareholders as it reduces over-investment by low talent managers as well as it provides an incentive for the board to invest ex-ante in learning about the manager’s true talent level. However, we also find that board power comes at a cost. When the board has the power over decisions, but is only partially informed about the manager’s true talent, then it may optimally decide to use its power to reject the manager’s investment idea. This action can be detrimental if the CEO is in fact a highly talented “superstar”.

Thus, our model highlights a major cost of board power stemming from the fact that board power and board knowledge are not the same. In particular, if the CEO is highly talented and is uniquely skilled at making highly innovative investments then being second-guessed and restricted by a board that does not understand the value of the investment can be harmful to shareholders. The problem is that a well-meaning board can be unaware that their CEO is indeed truly talented and hence this board may, unintentionally, stand in the way of the superstar CEO from fulfilling her high potential.

Modeling this basic tradeoff generates several novel implications regarding how board power affects the likelihood of (efficient) innovative investment, how it affects the decision of the board to become informed and hence the decision to replace the manager, and how it affects overall firm value. First, we show that greater board control can harm investments since powerful boards will tend to reject the investments suggested by CEOs who have a tendency to over-invest. While this is the direct effect of board power we also show that board power will increase the board’s incentive to learn about the CEO’s talent level and hence increase the likelihood that the efficient innovative investment will be made. The paper discusses the conditions under which this second indirect effect dominates the first direct effect leading to an overall positive relation between board power and efficient investment.

Second, our model has implications for how managerial turnover relates to measures of good corporate governance (i.e., board control). In particular, several
empirical papers (e.g., Weisbach 1988, Denis et. al. 1997, Huson et. al. 2001, and Jenter and Lewellen 2010) use a low sensitivity of CEO dismissal to negative stock returns as a measure of poor governance. Our model demonstrates that this argument is only partially true. In our model, the board may or may not reject the manager’s decision to make the investment. One can view the decision of the board to overturn the manager’s investment choice as equivalent to firing the manager. Hence, we show that board power has two effects on whether or not the manager is “replaced” following a negative signal from the market. The first direct effect of board power is that it increases the probability that the board will replace (overturn the manager’s investment choice) the manager following negative market information. The second indirect effect is that higher ex-post board power also increases the board’s ex-ante incentive to collect information. Thus, board power increases the chance that the board will have private information about the manager’s talent, which in turn will decrease the likelihood that the board will respond to the noisy signal that the market provides. The paper then characterizes the conditions under which greater board power will result in boards that ignore a negative market signal.

Finally, the model generates implications for when a strong board increases overall shareholder value and when a strong board will end up decreasing the value of the firm. For example, we find that greater board power increases firm value when the board’s costs of producing information are low and when the market signal is less noisy. In contrast, we find that greater board power reduces firm value when the ex-ante pool of CEO candidates is of higher quality.

One immediate policy implication from our model concerns the theoretical assessment of the benefit of the legislation that all boards must have a majority of outside directors. The intent of the legislation was to give boards greater power over decisions, but as a consequence the legislation forced firms to search for new directors that were, arguably, less knowledgeable and less experienced relative to incumbent directors. Our model would suggest that this is the exact combination that results in a decrease in firm value. Namely, increasing the power of the board should be done in combination with an increase of the board’s (average) talent level.
Our work relates to the recent literature debating whether legislators should try to further increase the power of the board. For example, Bebchuk (2005) noted that the control of the board over decisions is best even when board members are uninformed. We examine this argument more closely and show the conditions under which it is true.

Two related theoretical papers that have also analyzed the potential benefit of weaker boards and greater managerial control, but which use a different tradeoff than ours, are Adams and Ferreira (2007) and Harris and Raviv (2008). These papers use a cheap-talk model based on Crawford and Sobel (1982) to investigate a setting where both the CEO and the board have value relevant information that is critical for investment. These two papers analyze how board power affects the communication process between the board and the manager.

Adams and Ferreira (2007) consider a board that needs the private information of the CEO in order to collect information on what is the optimal investment. They show that a weaker board can be better for shareholders because weak boards will motivate the CEO to share her private information. The main result of the paper is that weak boards will be better able to gather information and hence that shareholder value would be better enhanced if the monitoring role and the advising role of the board could be done separately. The result in Adams and Ferreira (2007) that weak boards will be more informed than strong boards is in sharp contrast to our finding that stronger boards will always be better informed. The main reason for this key difference is due to the different agency problems of the two papers. While in their model the manager needs the board’s information about the state of nature to make better investments, in our setting the agency problem is over-investment and the manager is always informed.

In addition, we model a world where not all managers over-invest but only the low talent managers do. Thus, in our setting, board power helps solve a problem due to uncertainty about managerial skill and in their setting board power helps solve a problem due to uncertainty about what should be the optimal investment given the uncertain state of nature. The different focus of our paper also results in a different set of empirical implications that are not present in their work. For example, we find that a more powerful board will have a stronger incentive to collect information about managerial type and that this may actually result in a lower sensitivity between turnover and market stock returns.
As another example, we find conditions under which friendly boards (i.e., powerful
manager) lower the efficiency of investments.

Harris and Raviv (2008) also consider the case where board input helps
investments and analyze the optimality of having insiders on the board. Similar to Adams
and Ferreira (2007) they find that insider control may be optimal because it allows for
better use of the managers information. In addition, they find that insider control also
provides greater incentives for outsiders to become informed.\(^1\) This, again, is in contrast
to our key result that giving the board greater control also raises their incentive to become
informed.\(^2\)

In sum, our model analyzes an alternative channel through which managerial
power affects the economic environment. We show that board power can also be costly
even when there is no benefit for communication between the board and the CEO.
Managerial power will discourage the board from collecting information because the
board will be less able to use this information ex-post. Our model also stresses the
importance of managerial talent and of noisy public information in determining the
optimal allocation of power to a board.\(^3\) In addition, we aim to highlight the notion that
not all managers will harm shareholder value if left to invest on their own and analyze the
implications of this environment.

Finally, Song and Thakor (2006), analyze a board that plays an advisory role to
the CEO when both the board and the CEO have career concerns and hence may wish to
bias their reports to each other. They show that the CEO will prefer a weaker board
during economic upturns.

The rest of the paper is organized as follows: In section 2 we describe the model.
Section 3 discusses the equilibrium and derives the main results of the paper. Section 4
concludes.

\(^1\) In related work Harris and Raviv (2010), build on the work of Aghion and Tirole (1997) and explore the
issues of formal versus real authority in a setting where shareholders can delegate decision power to
management.

\(^2\) This aspect of our paper relates to Burkart Gromb and Panunzi (1997) who showed how monitoring by a
large shareholder discourages the manager from making firm specific investments. Burkart Gromb and
Panunzi (1997), however, do not consider the question of the optimal allocation of control.

\(^3\) There is also a related literature looking at the dynamics of board power (e.g., Hermalin and Weisbach
1998) and at the determinants of board member characteristics (e.g., Ruheja, 2005).
2. Model

We model an all equity firm that has a manager who is in charge of making investment decisions and a board of directors who needs to approve any major investment decision brought forth by the manager. Initially, at t=0, the firm sets its corporate governance structure (i.e., the allocation of board and CEO power) by setting the firm’s charter. The board then hires a CEO whose type is uncertain. At t=1 the CEO learns about an investment opportunity whose value depends on her type. The CEO decides on whether or not she wants to present the investment opportunity for board approval. If the manager decides to present the investment idea to the board, at t=2, then the board collects information about the manager’s investment idea. In addition, the market also generates a noisy signal about the project. Based on its information and based on whether it has control the board either approves or rejects the project. Finally, at t=3, cash flows are realized and the firm is liquidated. The sequence of events in the model is depicted in Figure 1.

Below we provide further detail of the model and the key players.

2.1 Manager

The manager is characterized by her talent level, which impacts the expected value of her investment project. The manager is hired from the general pool of managers which have a talent distribution, $\theta$, with $\theta \sim f(\theta)$. The manager of type $\theta$ identifies an investment with an expected cash flow of $k\theta$ where $k > 0$. If the manager decides not to invest in this project she can generate a cash flow of $R$.\(^4\)

In addition to generating cash flows we assume that investing in the project yields the manager with additional utility denoted by the parameter, $B$. We assume that $B > 0$ and interpret this parameter as reflecting the manager’s private benefit from investing.\(^5\) Therefore, the agency problem facing the board is a standard over-investment problem.

The manager is assumed to be risk neutral and to own a fraction $w$ of the firm’s equity. Hence the manager decides on whether or not to make the investment proposal to

\(^4\) One can think of the investment as an innovative long-term investment in the sense that its cash flows strongly depend on the talent level of the manager. For a more explicit model of innovations see, for example, Holmstrom (1989) and Manso (2012).

\(^5\) The main results of the paper would follow for the case of an under-investment problem in which $B<0$. Hence, we focus on the over-investment case to simplify the exposition of the paper.
the board by comparing her utility from investing, \( wk\theta + B \) to her utility from not investing, \( wR \).

2.2 Board of Directors

The board of directors is set with the task of reducing the agency costs associated with the over-investment problem. Since the investment project is a significant one it requires the potential approval by the board. The role of the board is thus to collect information about the project and, if it has the power, approve or reject the investment decision based on maximizing shareholder value. By assumption, there is no conflict between the board and shareholders.

2.2.1 Information

The board is initially uninformed about the talent level of the manager and hence about the expected value of the proposed project. If the board was fully informed and if the board had control over decisions then it would approve the investment if the manager was sufficiently talented and reject the investment if the manager was of sufficiently low talent and the expected cash flows from the project were less than \( R \).

The problem then arises because it is costly for the board to collect information. In particular, we assume the following information technology:

**Assumption 1:** If the manager approaches the board with an investment project the board selects to make an effort, \( e \), to collect information about the project. After making this effort the board learns the true expected cash flows from the project, \( k\theta \), with probability \( e \) but with probability \( 1 - e \) the board learns nothing. The cost of becoming informed is, \( C(e) = \frac{1}{2} ce^2 \).

In addition to the private information collected by the board we also assume that the board can observe a public signal that is produced in the market following the managers announcement about her intended project (but before it is finally approved or

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6We assume that the manager’s stake in the firm is given exogenously. The problem becomes highly non-linear if we assume that the contract is endogenous and hence the comparative static analysis becomes more difficult. Since our focus is on the issue of board power we abstract away from this added complexity.
rejected by the board). The board can use this information if it finds it helpful in making a decision. In particular, we assume that the market generates a signal $S$ with the following property,

**Assumption 2:** For any manager type, $\theta$ the market signal $S$ is distributed based on $\xi(S \mid \theta)$ where the distribution $\xi(S \mid \theta)$ satisfies the monotone likelihood ratio property with respect to, $\theta$.

Assumption 2 above simply states that the market signal is a noisy but informative measure of the quality of the manager and that a higher signal is more likely to come from a high talent manager that has an investment project with higher expected cash flows.

The market signal can potentially help the board make a decision about whether or not to approve the project. If the board’s investigation reveals the true type of the manager then the board does not need the information that is revealed in the market. If, however, the board’s investigation leads to no information then the board can use the market’s noisy signal to help make a decision.

Recall that the basic tension in the model is that a fully informed board prefers to allow the high talent manager to invest in the project and prefers that the low talent manager will not make this investment as it harms shareholders. The uninformed board, in contrast, may prefer that the manager does not invest if it makes an assessment that there is a sufficiently high probability that the manager is a low talent one. Hence, collecting information about the manager’s type is valuable as this may reduce the cost of over-investment.

### 2.3 Governance

The governance of the company is defined based on the parameter $g$ which represents the probability that the power to approve or reject the investment decision is at

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7 For papers looking at the informational role of stock prices see, for example, Dow and Gorton (1997), Goldman (2004), and Goldstein and Guembel (2008).
8 Although in our model the board is either fully informed or uninformed relative to the market the results would follow through if we allow for a board that receives a signal that is less noisy than that of the market.
the hands of the board of directors. Due to the agency problem, some managers will come to the board with a recommendation to make a major investment that is not good for shareholders. Because the board’s objective is to maximize shareholder value this means that with probability with probability $1 - g$ the manager will have control over the board and hence will be able to get approval for her project regardless of its value to shareholders.

The definition of governance in our model can be interpreted as measuring the ability of the board to reject major investment projects that the manager brings for board approval. While in practice the manager decides on the smaller day to day investments without board approval, we view the importance of board governance as it relates to the board’s ability to influence very big investments that have a large impact on the strategic position of the firm. In our model the board can voice its opinion based on its assessment of the manager’s ability to manage this major innovative project. Note that we also interpret a rejection of the manager’s proposed project as equivalent to a decision to replace the manager. Therefore, we can later analyze how governance affects the probability that a manager will be replaced.

3. Solving the Model

In this section we solve for the equilibrium of the model and discuss the main results of the paper. The equilibrium involves: 1) a decision by the manager on whether or not to propose the investment project to the board, 2) the board’s optimal effort in collecting information about the project, 3) the decision by the controlling party on whether or not to accept the project conditional on their information, and 4) the ex-ante choice of the governance parameter, $g$, that maximizes firm value.

To solve the model we begin by first analyzing the set of investments that will be proposed by a manager and the ex-post investment decisions that will be made by the board and by the manager based on their information and based on whoever is in control. We then solve for the board’s optimal effort to collect information about the proposed project when taking these future investment decisions into account. Finally, we solve for the ex-ante level of board control (i.e., firm governance) that maximizes the expected value of the firm.
3.1 Ex-Post Investment Choice

The investment decision depends on the state of nature, which is defined by the manager’s type (and hence the expected value of the project), the board’s information, and the board’s power to approve or reject the proposed investment. Figure 2 summarizes the possible states of nature and the investment outcome in each state as discussed below.

Given the distribution of manager types, $\theta$, one can define two constants

$$\theta_{SH} \equiv \frac{R}{k} \quad \text{and} \quad \theta_{M} \equiv \frac{wR - B}{kw}.$$  

These two constants partition the space of manager types into three regions. For all $\theta \geq \theta_{SH}$ shareholders (i.e., the board of directors) and the manager agree that making the investment is the best course of action because in this region, $k\theta \geq R$. For all $\theta \leq \theta_{M}$, the manager and the board agree that the best choice is not to make the investment because in this region, $wk\theta + B \leq wR$. Note that, in this region the manager’s ownership stake will give her the incentive to reject the project on her own and not even bring it up in front of the board. In this sense the equity stake of the manager can be thought of reducing the agency costs of over-investment.\(^9\) Finally, for all $\theta_{M} < \theta < \theta_{SH}$ the manager and the board would disagree on the best course of action.

While the manager would prefer to make the investment the board would prefer that the investment not be made because, $wk\theta + B > wR > wk\theta$. Thus, this region represents the states of nature in which the agency problem of over-investment has a real cost to shareholders.

Before analyzing the possible investment decision outcomes it is useful to define the following quantities.

**Definition 1:** Let $P_H$ be the probabilities that $\theta > \theta_{SH}$ and let $P_L$ be the probability that $\theta < \theta_M$. Furthermore, define $\pi_H$ by $\pi_H = E(k\theta | \theta > \theta_{SH})$ and $\pi_L$ by $\pi_L = E(k\theta | \theta_{M} < \theta < \theta_{SH})$.

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\(^9\) In this paper we abstract from any consideration of the optimal contract. Recent papers that focus on optimal contracts as a way to limit over-investment costs include, for example, Almazan and Suarez (2003), and Inderst and Mueller (2010).
From our above discussion it follows that the expected profits from the investment project satisfy, \( \pi_L < R < \pi_H \). Consider the possible states of nature and the investment decision that will be chosen in each state. In the case where the manager learns that \( \theta < \theta_m \) she will simply reject the project upfront rather than bother bringing it to the board of directors. This is the benefit of her having an equity stake in the firm. For all other cases where, \( \theta \geq \theta_m \), the manager will present the investment to the board and will recommend that the investment should be made. In these cases the final decision will depend on the power of the board and on the information that the board has. If the board has the power to reject the project and is informed it will decide to accept the investment if it learns that \( \theta > \theta_{sh} \) but reject the investment if it learns that, \( \theta < \theta_{sh} \).

If the board has power but was not able to learn from its private investigations about the quality of the project then the board will rely on the information that is provided by the market. In this case the board will observe the market signal and use it to update its assessment of the manager’s talent level and hence of the viability of the investment.

Given Assumption 2 it is easy to verify that there exists a constant \( S_H \) such that for \( S \geq S_H \) we have that \( E(k\theta | S) \geq R \) and for \( S < S_H \) we have that \( E(k\theta | S) < R \). It will be further helpful to define that following variables,

**Definition 2:** Define by \( h \) the probability that \( S \geq S_H \) conditional on \( \theta > \theta_{sh} \). In addition, define by \( \alpha = \frac{P_H}{1 - P_L} \) the probability that an investment that is proposed to the board is indeed good for shareholders. Hence, \( \alpha h \) represents the probability that the market signal is high and that the project is a good one while \( \alpha(1-h) \) represents the probability that the market signal is low conditional on the project being good.

Similarly, we can define by \( l \) the probability that \( S < S_H \) conditional on \( \theta_m < \theta < \theta_{sh} \). Thus, \( (1-\alpha)l \) represents the probability that the market signal is low and that the proposed project is bad for shareholders.
A board that has to rely on noisy market information in order to make decisions will allow the manager to make the investment if the market signal is sufficiently high and will reject the investment if the signal is sufficiently low. Using this informative but noisy signal will lead to two types of errors: sometimes allowing bad projects to proceed and sometimes rejecting good projects.

The last case to consider is the one in which the manager has control. In this case, regardless of the information that the board has, the manager will choose to invest in the innovative project as long as \( \theta \geq \theta_M \).

### 3.2 Endogenous Information

The board understands the investment outcomes in each future state of nature and hence the board has to choose ex-ante the optimal effort put in to collecting information. The board’s effort decision comes only after observing a manager coming with a request to approve her project. In this case the board will choose an effort level, \( e \), to maximize shareholder value as follows,

\[
\max_e \left\{ e \alpha \pi_H + (1 - e) R + (1 - e) g(\alpha h \pi_H + \alpha (1 - h) R + (1 - \alpha) l R + (1 - \alpha)(1 - l) \pi_L) + \\
+ (1 - g) (\alpha \pi_H + (1 - \alpha) \pi_L) - C(e) \right\}
\]  

The above profit function reflects the investment inefficiencies that come about in the model. The first is that the manager will sometimes want to make the investment even if this is not optimal for shareholders. This will create inefficiencies when the manager proposing the project is a low talent manager but the board has no power to stop her from investing. This will also create an inefficiency, but to a lesser extent, when the board has power but is uninformed and has to rely on the noisy market signal.

The second inefficiency arises when the manager is a high talent manager and hence she has a good investment project. In this case the inefficiency will come about when the board has the power to accept or reject the project but the board was unable to learn from its investigations whether or not this project is indeed good. Being uncertain about the manager’s true type the board will then act based on the market signal which
with some probability will lead to inefficiently reject a good project. Note, however, that although this action will lower firm value, it is still the board’s best choice given its (lack of) information.

One example for this type of inefficiency would be the situation in which the board hired a highly talented visionary CEO (e.g. the late Steve Jobs) to run the firm, but was unsure whether or not he was indeed a genius. Any intervention by the board in the decisions made by this manager will limit the profitability generated by the manager. Intervention, of course, would be the right decision by the board if it observes negative market signals (e.g. Shai Aggasi former CEO of Better Place) since the board may be concerned that the manager is leading the firm in the wrong direction.

**Lemma 1** The optimal information collection effort by the board of directors is given by the following,

\[
e^* = \frac{G}{c} \left[ \alpha (1-h)(\pi_H - R) + (1-\alpha)(1-l)(R - \pi_L) \right]
\]  

**Lemma 2** The board’s incentive to become informed about the quality of the project increases with its power, and with the expected cash flows generated by the investment of high talent manager, but decreases with the probability that the signal generated by the market is informative and with the expected cash flows generated by a project of a low talent manager.

Lemma 1 describes the optimal effort to collect information. The board decided on how much information to collect based on its power, the availability of the (free) noisy information from the market, and the inefficiencies created from being uninformed.

In Lemma 2 we describe some of the basic comparative static results concerning the incentive of the board to become informed. First, Lemma 1 and Lemma 2 show that the board’s incentive to become informed increases with the governance of the firm (i.e., with board power), \( g \). This result is due to the fact that the board’s information is only useful if the board has power to use it while its information has no value if the board ends up with no power to decide on which investment to make. Therefore, a board that expects to have power will have a higher incentive to learn about what should be the right course of action. Note, that this result is in sharp contrast to Adams and Ferreira (2007) and Harris and Raviv (2008) who argue that higher board power will result in less informative
decisions because a powerful board will discourage the manager from sharing her information.

Second, the two lemmas show that the board’s incentive to collect information is higher if expected cash flows from a project run by a high talent manager are higher and if the expected cash flows from a low talent manager are lower. The intuition for this result is that becoming informed helps the board limit investments by low talent managers and avoid limiting investments made by high talent managers. Thus, as the costs associated with making these mistakes increase the benefits of becoming informed increase as well.

Third, effort to collect information decreases with the probability that the market signal is informative, as measured by higher values of $h$ and $l$. The intuition here is that with a more informative and less noisy market signal the board does not have to exert costly effort to collect its own private information.\(^\text{10}\)

### 3.3 Board Power and Investment

One benefit of explicitly modeling the power of the board to approve or reject major investments is that we can now analyze the relation between board power and various decisions made by the firm. For example, we can analyze how board power affects the firm’s incentive to make large innovative investments.\(^\text{11}\)

**Definition 3**: Let $Q$ be the probability that a firm makes an investment. Then,

$$Q \equiv (1 - P_e) [1 - g \{1 - (e^\alpha + (1 - e^\alpha) [\alpha h + (1 - \alpha)(1 - l)])\}]$$ (3)

Investments are a key ingredient of any profitable company. From Equation (3) we see that board power, $g$, has a direct negative effect on the likelihood of making an

\(^{10}\) In this sense the information in market prices impacts the information that the board collects. For a more general approach analyzing these types of complementarities see Bond, Goldstein, and Prescott (2010).

\(^{11}\) Our paper looks at the impact of board power on investments, while others have looked at the impact of different aspects of the governance structure on investments. For example, Tirole et al (2011) analyze the impact of large shareholders on innovation, while Edmans (2009) and Goldman and Strobl (2013) analyze the impact of large shareholders on long-term projects.
investment as $\frac{\partial Q}{\partial g} < 0$. The reason for this is that the board of directors will try to limit investment to those cases where the board is sufficiently certain that the proposed investment is managed by a high talent CEO.

However, as can be seen from Equation 2 and Equation 3, board power also has an indirect effect on $Q$ through its impact on the board’s incentive to collect information. In particular, board power increases the incentive of the board to become informed which may result in more investment activity. Lemma 3 below characterizes the condition under which a more informed board will lead to increased investments.

**Lemma 3**: A higher investment in information collection will increase the probability that the firm will invest if and only if the following condition is satisfied,

$$\alpha \left(1 - \alpha \right) > 1 - l \left(1 - h \right)$$

Although higher board power tends to decrease investments, condition (4) in Lemma 3 shows that a more informed board will tend to increase the likelihood of investment approval and hence suggests that more powerful boards may result in an increased level of investments. This can happen because a more powerful board will also (endogenously) become more informed about the manager’s talent and hence the value of her investment idea.

Intuitively, a more informed board (due to greater board power) will lead to a higher likelihood that the firm will make the investment if the probability that the manager is a low type and the market signal is high, $(1 - \alpha)(1 - l)$, is lower than the probability that the manager is a high type and the market signal is low, $\alpha(1 - h)$. In this case an uninformed board will tend to allow more investments than an informed board.

So far we have shown that a powerful board will sometimes discourage investments because it will take away the discretion that a manager has to invest. However, we also showed that board power may increase investment in innovation, where managerial talent is a critical input to the success of the project, since stronger boards will also have a greater incentive to put more effort to become informed about the
manager’s talent level and the project at hand.\textsuperscript{12} Note that the lemma above considers total investments, which can be both good and bad for shareholders. In the lemma below we consider the impact of board power on \textit{efficient} investment; that is investment that maximizes shareholder value.

\textbf{Definition 4:} Let \( Z \) be the probability that a firm makes the investment when this is optimal for shareholders. Then,

\[ Z = (1-P_L)[\alpha - \alpha(1-e^*)g(1-h)] \tag{5} \]

Efficient investment can only occur when the CEO is sufficiently talented, as indicated by \( \theta > \theta_{sl} \). In this case the only time the investment project will be rejected is when the board has power but was unsuccessful in obtaining private information about the manager and her project. In that situation the board will rely on the market signal to decide whether or not to reject the project.

From Equation (5) above we see that a more powerful board will actually end up decreasing the likelihood that good investments will be accepted as, \( \frac{\partial Z}{\partial g} < 0 \). This happens, again, because a powerful board helps in reducing inefficient investment when the manager proposes a project and she is a low type, \( \theta_M < \theta < \theta_{sl} \), but at the cost of sometimes hurting efficient investment when the manager is a very talented one and hence has a great project. In addition, we need to consider the fact that the board also optimally chooses how much effort to put into becoming informed. It is clear that \( \frac{\partial Z}{\partial e} > 0 \) which means that a more informed board will result in a higher likelihood of efficient

\textsuperscript{12} We focus here on the probability of making an investment rather than the level of investment, which is not modeled here. Our results generalize to a model where the invested amount is chosen as part of the maximization problem. Namely, if we let the amount invested in innovation be endogenously determined we can show that board power has a negative direct effect on innovation but a positive indirect effect through the incentive of the board to become informed. Thus, talented managers who face strong boards will optimally avoid making innovative investments for fear that the board will not be able to understand the hidden long-term value in these projects. This will be true unless the board is sufficiently informed. The model with endogenous investment level, however, complicates the analysis without adding much intuition.
investment. Lemma 4 below characterizes what is the total impact of governance (i.e., board power) on efficient investment.

**Lemma 4:** Increasing board power will lead to a higher probability of efficient investments by the firm if and only if the board’s effort to collect information is sufficiently high.

The intuition behind this result is that a governance system that gives power to the board over the CEO will only increase efficient investment if board power is also accompanied with a board that has the incentive and ability to become informed about the quality of the manager at the helm. In other words, an uninformed board that has power is actually very detrimental to the efficiency of investments. More broadly, we can interpret the result in Lemma 4 as indicating that strong board governance will lead to higher investment for large values of $e^*$ and will lead to lower levels of innovation if $e^*$ is low. Thus, the empirical implication of the lemma is that any parameter that increases $e^*$ will increase the probability that we are in a regime where governance and investment are positively correlated. Corollary 1 below summarizes some comparative static results that lead to specific empirical implications.

**Corollary 1:** Increasing board power results in a higher probability of efficient investment whenever it is less costly for the board to become informed (lower $c$), whenever public information is less reliable (lower value of $h$ and $l$), whenever the expected cash flows generated by the high talent managers are higher, high $\pi_h$, and whenever the cash flows generated by low talent managers are lower, low $\pi_L$.

### 3.4 Turnover Following Negative Market Signal

We consider the possibility that the board rejects the manager’s proposed investment. If we interpret the decision to reject the manager’s investment project as equivalent to firing the manager then we can analyze how board governance impacts turnover decisions.

In this sense it is especially instructive to follow the empirical literature (e.g., Weisbach 1988, Jenter and Lewellen, 2010), which has used CEO turnover events in
order to measure what a good governance board should look like. In particular, this literature has focused on measuring the responsiveness of the board to negative signals from the market. The interpretation has been that boards that are not responsive to negative market signals represent cases of poor corporate governance. For example, if the relation between turnover and (negative) market signal is lower for large boards then the conclusion should be that large boards are not good for shareholder governance.\textsuperscript{13}

To analyze the relation between governance and how responsive the board is to negative market signals we first define the following measure.

**Definition 5:** Let $\text{RESP}$ be the probability that we observe a board rejecting the manager’s proposal following a low (“negative”) market signal. Then we have,

$$
\text{RESP} = \frac{\text{Prob} \{ \text{reject project} \mid S < S_H \}}{\text{Prob} \{ S < S_H \}}
$$

A calculation of the probabilities of the relevant outcomes from Figure 1 yields the following result,

$$
\text{RESP} = g \frac{\alpha(1-e^\alpha)(1-h) + (1-\alpha)l}{\alpha(1-h) + (1-\alpha)l}
$$

From Equation 6 we can see that the direct effect of better governance (higher $g$) on the board’s responsiveness to a low market signal is positive as $\frac{\partial \text{RESP}}{\partial g} > 0$. This is indeed what has motivated the empirical literature, which investigated CEO turnover events, to conclude that better governance should manifest itself in a higher sensitivity of CEO turnover to stock price declines.

What we want to emphasize is an additional channel through which governance and board power impacts the decision of the board to respond to negative market

\textsuperscript{13} While large boards have been viewed as bad for shareholders, Boone et. al. (2007) analyze empirically the determinants of board structure and find evidence consistent with the idea that different board structures can maximize shareholder value in different situations. Their empirical results are consistent with our general claim that the optimality of board power is endogenous and hence a “one size fits all” approach is problematic.
information. From Equation 6 we can also observe that, \( \frac{\partial \text{RESP}}{\partial e} < 0 \). This means that a more informed board will actually be less responsive to negative market signals. The intuition here is that a more informed board will have better information than the market about the manager’s skill level and hence will have less use for the markets information. More generally, an informed board will put less weight on any (new) market information regarding the quality of the manager and her project. This will imply that the board will be less likely to replace the manager after a bad market signal. When the private information collected by the board is determined endogenously we argue that better governance (as indicated by a board with more power) may lead boards to ignore negative market signals. The lemma below describes when this will occur.

**Lemma 5:** Boards with more power over decisions will not always exhibit a higher probability of replacing the manager following a negative market signal. Stronger boards will be associated with a higher value of RESP if and only if the equilibrium level of information collected by the board is sufficiently low as indicated by,

\[
e^* \leq \frac{1 + \frac{(1-\alpha)l}{\alpha(1-h)}}{2}
\]

**Corollary 2:** As long as \( \alpha \) is sufficiently high as indicated by, \( \frac{\alpha}{1-\alpha} > \frac{l}{1-h} \), then the relation between board power and RESP will be negative when the cost of collecting information, \( c \), is sufficiently low, when board power, \( g \), is sufficiently high, and when expected cash flows generated by high talent managers, \( \pi_H \), are sufficiently high.

Lemma 5 indicates that we should expect more powerful boards to be more responsive to negative signals from the market only if their efforts to privately collect information are low. Corollary 2 describes some specific scenarios when this will be the case. For example, the board will be less informed if it has high costs of privately collecting information, if it expects to have little power, or if the cost of limiting the investment of a high talent manager is low. A more general interpretation of our findings is that boards that decide to ignore negative market signals could be good boards that
have very strong (endogenous) priors about the quality of the manager and her suggested project.

3.5 Firm Value

The inefficiencies created by the agency problem is that all managers, above a certain talent threshold, want to proceed with the investment but that, from shareholders perspective, only the very talented managers should. If we could achieve the first best investment scheme we would observe a firm value of,

$$V^{FB} = P_L R + (1 - P_L)[\alpha \pi_H + (1 - \alpha)R]$$  \hfill (8)

The board of directors can potentially increase firm value by collecting information and rejecting the investment whenever it identifies that the manager is a low talent manager and the board has power. This comes at the cost of collecting information, at the cost of making a mistaken decision when not fully informed, and at the cost of, at times, not having the power to change a bad decision by the manager. The above costs and benefits result in the following expression for the value of the firm under the equilibrium actions,

$$V(g, \alpha, ...) = P_L R + (1 - P_L)H(g, \alpha, e^*)$$  \hfill (9)

Here $H(g, \alpha, e^*)$ is the value function achieved from the maximization in Equation (1). Given this second best firm value, which is a function of $g$, we have the following result.

**Lemma 6:** The level of board power that maximizes firm value is at one of the two extremes. It is optimal to either give the board maximum power, $g^* = 1$, or give the manager full power, $g^* = 0$.

As Lemma 6 indicates firm value is maximized either when the board has full power or when the manager has full power. It is first helpful to look at the first order condition, which due to the envelope theorem ($\frac{\partial V}{\partial e} \big|_{e=e^*} = 0$) simplifies to,
\[
\frac{1}{1-P_L} \frac{dV}{dg} = \alpha (1-e^*)[(h\pi_H + (1-h)R - \pi_H) \\
+ (1-\alpha)(e^* (R - \pi_L) + (1-e^*)(IR + (1-l)\pi_L - \pi_L))]
\]

(10)

The first term is negative while the second term is positive which is consistent with the intuition that board power is beneficial to shareholder value if the manager ends up being of low quality (probability 1 – \(\alpha\)) but is detrimental to shareholder value if the manager is a talented “super star” who has a good project to invest in (probability \(\alpha\)) but who is facing an uninformed board.

Due to the above tradeoff we can derive some comparative statics on the costs and benefits of board power by comparing the value of the firm when \(g^* = 1\) to its value when \(g^* = 0\). This leads to the following expressions:

\[
V(g^* = 0) = P_L R + (1 - P_L)\{\alpha \pi_H + (1 - \alpha) \pi_L\}
\]

and

\[
V(g^* = 1) = P_L R + (1 - P_L)\{\alpha [e^{*g=1} \pi_H + (1-e^{*g=1})(h\pi_H + (1-h)R)] + \\
+ (1-\alpha)(e^{*g=1} R + (1-e^{*g=1})(IR + (1-l)\pi_L)) - C(e^{*g=1})\}
\]

(11)

Equation 11 shows the value of the firm under full manager power. In this case the board optimally decides to collect no information about the manager and hence this is the “no information” case. Here we see that the loss of value arises due to the fact that the weak board will allow low talent managers to make investments that do not maximize shareholder value.

Equation 12 shows the value of the firm when the full power over decisions is given to the board of directors. This can be termed the “high information” case because this is the case in which the board optimally collects information about the manager. Here, the loss of value (relative to the first best) arises because the board incurs a cost of collecting information and because the effort to learn about the manager’s skill level and the value of her project does not always result in a fully informed decision by the board.
We now define the variable $\Delta$ as

$$\Delta \equiv V(g^* = 1) - V(g^* = 0)$$

This is simply the difference in firm value between the case of high board power and low board power. In the lemma below we analyze under what conditions this difference becomes larger and under what conditions it becomes smaller. Since $\Delta$ can be either positive or negative we can interpret any variable that increase (decreases) $\Delta$ as increasing the states of nature where board (manager) power is optimal.

**Lemma 7**: The difference in firm value between a firm with a powerful board and a firm with a weak board, $\Delta$, is:

i) Decreasing in $\pi_H$, and in $\pi_L$.
ii) Increasing with the board’s ability to produce information, (low $c$).
iii) Increasing with the accuracy of information generated by the market, as measured by a higher $h$ and $l$.
iv) Decreasing with the fraction of CEOs who are of high talent, $\alpha$.

Board power is better for shareholders when the cash flows generated by the investment of either the high or the low talent managers are higher. The reason is that a more powerful board is more likely to step in and reject the suggested investment. Thus, when these cash flows are expected to be higher relative to the return to not investing, $R$, it is less valuable to give the board ex ante power.

When the board has greater ability to produce information it is more beneficial to allocate power to the board and away from the manager. This is because the decisions of an informed board are always better for shareholders than the decisions of the manager who may or may not chose to invest according to shareholders’ preferences.

The accuracy of market signals is also a key factor in determining the impact of board control on firm value. The implication from our model is that when market information is more accurate the benefit of allocating the board with power increases. This is because the cost of giving the board power is that the board may end up relying on market information in order to make the best investment decision. Hence, the more accurate is the information from the market the higher is the value created by the board. Note that in the extreme case where market information is fully accurate about
managerial type, and hence about the value of her investment, we get back to the first best value of the firm under, \( g^* = 1 \).

The lemma also shows that as the probability that the manager is highly talented increases (\( \alpha \)) board power becomes more detrimental. The reason again is that the cost of allocating power to the board is in the fact that an uninformed board with power will tend to intervene too much and lower the value created by a high talent CEO. Thus, as the likelihood that the CEO is indeed a superstar increases allocating control away from the CEO becomes more costly. It is interesting to note that several papers looking at the trend in CEO compensation have argued that this trend can be explained by an increase in the talent of the top CEO’s (e.g. Murphy and Zabojnik 2004, Kaplan and Rauh 2010). Our model would then argue that this is exactly the time to reduce board power rather than increase it (unless the talent pool of the board is also on the rise).

4. Empirical Implications

In this section we highlight some of the empirical implications from the model that were discussed throughout the paper.

4.1 Efficient Investment

Our model provides some insight as to whether allocating more power to the board will result in greater efficiency of investments. The key implication from our model is that only a skilled and informed board will increase the efficiency of investments. Thus, for example, we expect that a board with a venture capitalist should have greater power and better investment outcomes. In addition, a powerful board will lead to better investments whenever the market is less able to produce informative signals. This could be the case, for example, when the investment projects are innovative or when they are long-term projects and when the board members are better skilled (than the market) at understanding the potential of these projects. Thus, we expect that in more mature industries powerful boards will result in less efficient investments as in these industries it is more likely to expect that the market will be well informed about proposed projects.
4.2 Board response to market information

Our model shows that a board that is not responsive to (negative) market signals does not always indicate bad governance. We discuss in the paper the context of a board that decides not to fire a CEO even when market signals suggest it. The more general implications from our model are that empirical analysis that looks at the responsiveness of the board to market signals should include, for example, controls for the ability of the board (its “c”) and how informative is the signal (variance of S). Boards with experienced board members that do not react to market signals that are noisy should then be interpreted as potentially smart boards rather than boards that are controlled by the CEO.

4.3 Firm Value

When it comes to maximizing firm value our model implies that firms with weaker boards will be less valuable when the overall quality of innovative investment is high. This could be, for example if a firm operates in an industry with high growth options that require skillful managers.

Since better talented managers increase firm value when the board is weaker, our model predicts that firms that attract a better pool of talent will also have weaker boards. For example, if younger firms can only attract less experienced lower talent managers then we would expect them to also have stronger boards.

Another implication is that in industries with less informative stock prices and where firm operations are less transparent we would expect firms with weaker board to be of higher value.

Finally, as the pool of CEOs becomes more talented we would expect the costs of board power to increase and hence to lower firm value. This could have time-series implications depending on whether or not the talent pool of CEOs (either in general or in a specific industry) has changed over time.

5. Conclusion

In this paper we highlight the costs and benefits of allocating control to the board. We show that allocating more power to the board induces the board to become more
informed and hence improves the decisions made by the board. However, we also show that board control may come at a cost in those situations when the board is not sufficiently informed and must rely on noisy market information when deciding on major investment decisions.

We characterize under what conditions board power is preferred over managerial power as well as what can be learned from analyzing the relation between board control and managerial turnover and between board control and efficient investments.
The firm establishes its governance, $g$, by setting its charter.

The board hires a CEO of unknown talent, $\theta \sim f(\theta)$.

CEO learns of investment with expected cash flows $k\theta$

CEO decides whether or not to propose the investment to the board of directors

If CEO proposes making the investment then the board makes effort $e$ to collect private information.

Board also gets a noisy public signal $S$ on the value of the project.

The party with power decides on rejection or approval of investment

Cash flows are realized.

Firm is liquidated.
Figure 2: Investment Decision following market information

<table>
<thead>
<tr>
<th>Manager Type $\theta &gt; \theta_{SH}$</th>
<th>Market Signal</th>
<th>Decision</th>
<th>Cash flows</th>
</tr>
</thead>
<tbody>
<tr>
<td>Board informed</td>
<td>Has Power</td>
<td>Any $S$</td>
<td>Invest</td>
</tr>
<tr>
<td>No Power</td>
<td>Any $S$</td>
<td>Invest</td>
<td>$k\theta$</td>
</tr>
<tr>
<td>Board uninformed</td>
<td>Has Power</td>
<td>$S &gt; S_H$</td>
<td>Invest</td>
</tr>
<tr>
<td></td>
<td>$S &lt; S_H$</td>
<td>Do not invest</td>
<td>$R$</td>
</tr>
<tr>
<td>No Power</td>
<td>Any $S$</td>
<td>Invest</td>
<td>$k\theta$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Manager Type $\theta_M &lt; \theta &lt; \theta_{SH}$</th>
<th>Market Signal</th>
<th>Decision</th>
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</thead>
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</tr>
<tr>
<td>No Power</td>
<td>Any $S$</td>
<td>Invest</td>
<td>$k\theta$</td>
</tr>
</tbody>
</table>

| Manager Type $\theta < \theta_M$ | Do not invest | $R$ |
References


Appendix

Proof of Lemma 1: From the maximization problem in Equation 1 we see that the first order condition characterizes a maximum. The optimum can be easily computed from solving this first order condition.

Proof of lemma 2: Given the value of $e^*$ in Equation 4 we can see that,
\[
\frac{de^*}{dg} = \frac{1}{c}[\alpha(1-h)(\pi_H - R) + (1-\alpha)(1-l)(\pi_L - R)]
\]
which is positive. One can also verify that \[
\frac{de^*}{d\pi_H} = \frac{g}{c}(1-h)
\] which is positive and that \[
\frac{de^*}{d\pi_L} = -\frac{g}{c}(1-\alpha)(1-l)
\] which is negative.
A more accurate market signal is obtained when $h$ and $l$ are higher. This can be seen from the fact that, \[
\frac{de^*}{dh} = -\frac{g}{c}[\pi_H - R]
\] and \[
\frac{de^*}{dl} = -\frac{g}{c}(1-\alpha)[\pi_L - R].
\] Now since the expected cash flows from the investment project of higher type managers are higher than $R$ and the investment cash flows from investments made by lower talent managers are lower than $R$ the proof follows.

Proof of Lemma 3: Based on Equation 3 we see that a more informed board will increase investments, \[
\frac{dQ}{de} = (1-P_L)\alpha[\pi_H - R]
\] is positive. This occurs if and only if \[
\frac{\alpha}{1-\alpha} \frac{1-l}{1-h} > 1.
\]

Proof of Lemma 4: Taking the derivative of the probability of efficient investment with respect to board power we have that,
\[
\frac{dZ}{dg} = \frac{\partial Z}{\partial e} \frac{de}{dg} \bigg|_{e=e^*} = (1-P_L)(1-h)\alpha\frac{de}{dg} \bigg|_{e=e^*} - (1-e^*)
\]
But from the equilibrium value of effort we can see that,
\[
\frac{g}{dg} \bigg|_{e=e^*} = e^*
\]
And hence,
\[
\frac{dZ}{dg} \geq 0 \iff \alpha[e^* - (1-e^*)] \geq 0 \iff e^* \geq \frac{1}{2}.
\]

Proof of Lemma 5: From Equation 6 we have that,
\[
\frac{dRESP}{dg} = \frac{\alpha(1-e^*)(1-h) + (1-\alpha)l - g\alpha(1-h) \frac{de^*}{dg}}{\alpha(1-h) + (1-\alpha)l}.
\]
Using the fact that \( g \frac{de^*}{dg} = e^* \) we can focus on the term in the numerator to yield,

\[
\text{Sign} \left[ \frac{d\text{RESP}}{dg} \right] = \text{Sign} \{ (1 - 2e^*)\alpha(1 - h)+(1-\alpha)l \} \quad \text{which is negative if and only if the}
\]
equilibrium effort level is sufficiently high, as indicated by the condition in the lemma.

**Proof of Lemma 6:** From the first order condition,

\[
\frac{dV}{dg} = \frac{\partial V}{\partial g} + \frac{\partial V}{\partial e} \frac{de}{dg} \mid_{e=e^*} .
\]

Based on the envelope theorem the second term equals zero at \( e = e^* \). This yields a first order condition of,

\[
\frac{dV}{dg} = (1 - P_L) \{ \alpha (1- e^*) \left[ (h\pi_H + (1-h)R - \pi_H \right] \\
+ (1-\alpha) \{ e^*(R - \pi_L) + (1-e^*)[lR + (1-l)\pi_L - \pi_L] \} \}
\]

Since \( e^* \) is linearly increasing in \( g \) we have that,

\[
\frac{d^2V}{dg^2} = (1 - P_L) \frac{de^*}{dg} \left[ \alpha (1-h)(\pi_H - R) + (1-\alpha)(R - \pi_L)(1-l) \right] .
\]

It is easy to verify that the second order condition is positive. This means that the highest value of the firm is achieved on the boundary, either when full power is given to the board or when full power is given to the manager.

**Proof of Lemma 7:** First note that for any variable, \( x \), we have that,

\[
\frac{d\Delta}{dx} = \frac{\partial \Delta}{\partial x} + \frac{\partial \Delta}{\partial e} \mid_{e=e^*} \frac{de^*}{dx} \quad \text{but due to the envelope theorem we know that} \quad \frac{\partial \Delta}{\partial e} \mid_{e=e^*} = 0 , \quad \text{so that}
\]

we only need to consider the partial derivative with respect to \( x \).

Thus,

\[
\frac{d\Delta}{d\pi_H} = \alpha (e^* \mid_{g=1} + h(1-e^* \mid_{g=1}) - 1) < 0 \quad \text{and} \quad \frac{d\Delta}{d\pi_L} = (1-\alpha)((1-e^* \mid_{g=1})(1-l) - 1) < 0 .
\]

This proves (i). We can also see that, \( \frac{d\Delta}{dc} < 0 \) which proves point (ii). Looking at the impact of the accuracy of the market signal we see that,

\[
\frac{d\Delta}{dh} = \alpha (1-e^* \mid_{g=1}) \alpha (\pi_H - R) > 0 \quad \text{and} \quad \frac{d\Delta}{dl} = (1-e^* \mid_{g=1})(1-\alpha)(R - \pi_L) > 0 \quad \text{which are both}
\]

positive. Finally, we have that

\[
\frac{d\Delta}{d\alpha} = e^* (\pi_H - R) + (1-e^*)[h\pi_H + (1-h)R - lR -(1-l)\pi_L] - (\pi_H - \pi_L) .
\]

Note that the first two terms are an average with weights \( e^* \) and \( 1-e^* \). Thus we need to show that this average is less than \( \pi_H - \pi_L \). By the definition of \( \pi_H \), we have that

\( \pi_H - \pi_L > \pi_H - R \) so the first term in the average is smaller. Now for the second term we have that \( \pi_H > h\pi_H + (1-h)R \) and \( \pi_L < (1-l)\pi_L + lR \) so that the second term in the curly brackets is also smaller than \( \pi_H - \pi_L \). Hence the derivative is negative.