The Private and Social Value of Capital Structure Commitment*

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Abstract

We analyze dynamic stationary models of capital structure, in partial and general equilibrium, when managers cannot commit to firm-value maximization. The model permits us to quantify both the private cost to firms of the commitment problem, and also the aggregate cost of its externality. Our setting encompasses time-varying economic and firm characteristics, as well as valuation under generalized preferences. The model provides an explanation for the procyclical use of unprotected debt: the private costs of non-commitment increase in bad times. Likewise, expropriation incentives rise when firm valuations are low. Hence, without commitment, leverage can be countercyclical. This dynamic amplifies the effect of excess debt on aggregate risk. A range of parameterizations suggests that the social cost of unprotected debt can be large. We present evidence supportive of the prediction that firms with unprotected debt increase their borrowing in bad times.

KEYWORDS: CAPITAL STRUCTURE, COVENANT VALUATION, GENERAL EQUILIBRIUM, SOCIAL COST OF CONTRACTING FRICTIONS

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1. Introduction

This paper examines the quantitative implications of noncontractible debt-equity conflicts in partial and general equilibrium. In particular, we compute and contrast the private and social costs of equity-maximizing leverage policies.

Following the financial crisis of 2007-2009, a broad consensus has emerged that frictions in corporate finance can have first-order effects on the macroeconomy. However, while there is a vast theoretical literature delineating firm-level effects of financial frictions, progress in incorporating its insights into macroeconomic models has been modest. Important questions remain about *which* frictions matter in aggregate, and *how much*.

To address such questions, corporate frictions need to be examined in a setting that has several non-trivial properties. First, to be closeable, the model must permit aggregation over the population of firms. Second, the model must include a realistic preference specification in order to speak to to issues of welfare. Third, the driving exogenous state variables need to be rich enough to encompass time-varying business conditions. Fourth, at the firm-level, the problem should include endogenous financial policies that reflect the friction being modeled. Fifth, to examine aggregate moments, the model needs to admit unique stationary equilibria. Last, solutions should have quantitatively reasonable implications for the dynamic properties of the main real and financial quantities in the economy.

Our study examines one particular friction in a tractable and transparent framework that includes all of these features. It has long been well understood that equity holders have incentives to increase firm debt to the detriment of creditors, and possibly even to the point of lowering overall firm value. An influential post-crisis paper, Admati, DeMarzo, Hellwig, and Pfleiderer (2013), highlighted (in the context of banks) the potential social costs of these incentives.[1] Do equity-maximizing policies, in fact, have quantitatively

[1] Specifically, the paper examines and rebuts numerous arguments advancing the counter-hypothesis that *reducing* bank debt below equity-maximizing levels would entail social costs.

important implications for aggregate risk and welfare? The current work presents (to our knowledge) the first formal investigation of this subject.

We build on the subsequent work of Admati, DeMarzo, Hellwig, and Pfleiderer (2018) and DeMarzo and He (2018) who have analyzed dynamic capital structure models in partial equilibrium, in which managers cannot commit (at time-zero, i.e., when a firm is first capitalized) to *not* alter debt levels in the future. These works contain numerous insights, foremost of which is that, indeed, absent commitment, equity holders will prefer to take on excessive debt, and will resist debt reductions even when the value creation from doing so would be substantial. Their settings, however, do not encompass the elements described above that are necessary to address our topic.

We therefore introduce a new formulation of the commitment problem that increases tractability on several dimensions. The model yields closed-form optimal policies for leverage and default and for prices of debt and equity. Our equilibrium concept is intuitive and yields unique stationary equilibia for a wide range of relevant parameterizations. We also allow firm and macroeconomic parameters to change over time. Our analysis is undertaken using generalized preferences, rather than risk-neutrality. This enables us to quantify, in a fully-specified asset-pricing framework, the valuation consequences of the lack of commitment in debt policy.

Most importantly, the tractability of our framework permits aggregation across firms, meaning that we are able to close the model, and thus capture the feedback from corporate decisions to marginal utility and discount rates, which, in turn, affect the optimal policies. Solving the general equilibrium allows us to quantify the social cost and the degree of externality stemming from lack of commitment.

In addition to the unconditional effects of increased debt levels, our solutions also permit us to offer insight into the implications for debt dynamics. Recent empirical work (Halling, Yu, and Zechner (2016), Johnson (2018)) documents countercyclical patterns in target leverage ratios and debt issuance, suggesting that firms have private incentives to shift to debt financing in bad times. In the context of our model, countercylicality induces further welfare costs, which we quantify.

While the general equilibrium analysis is the principal contribution of our work, the firm-level building blocks also contribute new insights to the line of research studying the determinants and value consequences of covenants in debt contracts. (See Roberts and Sufi (2009) for a survey of this research.) Our model offers a theory of how incentives to achieve commitment vary over time (within a firm) and across firms with different characteristics, and thus when and why covenant strictness may vary. Indeed, the increasing prevalence of "cov-lite" debt,[2] has drawn the attention of global financial regulators, suggesting again that there may be an important social dimension to the contracting problem.

Our main results are as follows.

In common with the works above, our model produces greater leverage in the absence of commitment. Comparison of the equity-maximization decision with firm value-maximization reveals that the distortion incentives scale inversely with firm valuation multiples (e.g., Tobin's Q). The primary dimensions of the trade-off problem (default costs and tax benefits) scale with firm value, but the marginal expropriation benefit does not. Intuitively, when the firm's underlying business is valuable, expropriating wealth from existing creditors is a low priority for managers. To investigate magnitudes, we compute the loss of firm value in numerical examples. The surplus that accrues to firms that can achieve commitment (e.g., through covenants) can be economically large, and agrees reasonably well with estimates in Matvos (2013) and Green (2018).

Next, we document that this surplus is countercyclical. This parallels our first finding. Intuitively, again, in good times the marginal benefit of achieving commitment is small. This observation provides an explanation for the empirically observed procyclical use of

[2]At the end of 2018, 79% of the \$ 1.17 trillion outstanding leveraged loans in the U.S. were classified by S&P as cov-lite. Cross (2019).

unprotected debt, which is sometimes thought to be driven by investor irrationality or agency incentives to "reach for yield." Here, however, there is simply less reason for firms to assume additional contracting costs when they issue new debt in good times.

Because expropriation incentives rise when firm valuations are low, it is also the case that firms without commitment sell relatively more debt in bad times. The first-order trade-off incentive of higher expected default costs almost always implies that firms with commitment sharply reduce leverage in bad times. The same applies without commitment, typically also producing optimal decreases in leverage. But these decreases are significantly smaller, and optimal policies sometimes actually call for increasing debt issuance.

Turning to general equilibrium, the overall higher leverage without commitment imposes real costs due to higher default rates and output volatility. We show that the social cost in terms of welfare loss significantly exceeds the private cost borne by owners of firms. In addition to the level of excess debt, the increased countercyclicality of debt in the absence of commitment provides a further level of amplification, which can be of the same magnitude as the level effect.

While recognizing the limitations of drawing precise numerical conclusions from a model that remains a stylized depiction of the macroeconomy, we show that our results are qualitatively robust across a range of parameterizations. In particular, the findings with respect to the degree of externality do not depend strongly on the preference specification. We thus verify the conjecture in Admati, DeMarzo, Hellwig, and Pfleiderer (2013) that equity maximization in capital structure may entail significant social consequences.

Finally, we take the model's new testable implication to the data. Using covenant-based measures of commitment at the firm level, we present empirical evidence supportive of the hypothesis that firms with unprotected debt increase their borrowing in bad times, when economic growth is low or when valuation multiples are small. Back-of-the-envelope calculations suggest that if all debt were protected, the effect on aggregate leverage variability would be non-negligible. Besides contributing to the empirical literature on debt covenants, the findings support the assertion that the theoretical mechanisms in our model may have important effects both at the firm and aggregate levels.

1.1 Relation to Literature

Our paper extends the tractable stochastic environment introduced in Johnson (2018). That work is also concerned with the social cost of corporate debt decisions, and analyzes a general equilibrium trade-off model versus an alternative formulation embedding moral hazard. The form of the debt contract in that paper precludes commitment issues, however. So it does not address the potential real effects studied here.

Commitment problems in debt policy decisions were first formally modeled in Bizer and DeMarzo (1992) in the context of sequential borrowing. Other work has analyzed the lack of commitment on debt maturity. For example, Brunnermeier and Oehmke (2013) show that lack of commitment in maturity structure leads to shorter average maturity of debt because a firm has an incentive to shorten one creditor's debt contract to expropriate other creditors. In response, other creditors will demand short-term debt contract as well.[3]

On the leverage dimension, as discussed above, Admati, DeMarzo, Hellwig, and Pfleiderer (2018) have shown, in a simple two-period setting as well as numerically illustrating in a dynamic setting, that lack of commitment for future capital structure policies has profound impact on firms' leverage dynamics. Without a commitment device, shareholders always want to increase leverage following good performance, even when new debt is junior to old debt. Equity-maximizing managers are never willing to reduce leverage no matter how large the potential increase in overall firm value would be. The mechanism is that existing creditors would capture all the firm value enhancement that leverage reduction achieved.

[3]See also He and Milbradt (2016).

DeMarzo and He (2018) analytically derive the leverage dynamics of a firm without commitment, with a general cash flow process in continuous time.[4] They analyze a "smooth" equilibrium where leverage changes are gradual (of order dt). They find that shareholders never actively reduce leverage. They also speak to the effect of no-commitment on debt maturity structure choice as well as the mutual reinforcement between a firm's investment strategy and financing strategy if it faces commitment issues on both dimensions. Their model thus delivers rich implication about the effect of non-contractibility on firms' financing as well real decisions.[5]

More broadly, the present work belongs to an emerging literature examining the macroeconomic implications of contracting problems in general equilibrium.[6] Important contributions include Cooley, Marimon, and Quadrini (2004), who examine the inability of lenders to enforce repayment from entrepreneurs. Their model implies that limited contractual enforceability amplifies aggregate shocks through the cyclicality of entrepreneurs' hold-up power. In Levy and Hennessy (2007) countercyclical leverage arises from the need to maintain high managerial ownership in bad times to solve agency problems. Lorenzoni (2008) models two-sided limited commitment by lenders (households) and borrowers (entrepreneurs), in a three-period economy. As in our model, inefficient liquidation leads to welfare losses. In contrast, Gale and Gottardi (2015) model a social benefit to debt as default allows efficient firms to benefit by buying bankrupted firms' assets at fire-sale prices. Their equilibrium features too little, rather than too much debt,

[4]Our specification is not nested in that of DeMarzo and He (2018) because they do not allow the possibility of jumps into the default region. See their footnotes 15 and 17. We provide a full comparison of our (partial equilibrium) findings and theirs in Section 2.3.2.

[5]It is perhaps worthwhile to distinguish the contracting problem studied in these papers (and ours) from a separate literature that investigates the implications for financial policy of the principle-agent problem when managers maximize neither firm nor equity value, but rather their own private value. This problem is modeled in Morellec (2004). There, the manager's private value is maximized with inefficiently *low* leverage.

[6]There is, of course, an extensive literature on the role of financing frictions in business cycles. See Brunnermeier, Eisenbach, and Sannikov (2013) for a recent survey. Here we are distinguishing models wherein capital structure policy is an endogenous outcome under incomplete contracts, rather than being driven by exogenous supply constraints.

resulting in underinvestment and imposing a welfare cost.

To summarize, this study contributes both new tools and new insights towards the effort to examine corporate finance problems and macrofinance outcomes within the same setting. We quantitatively evaluate the implications of debt-equity contracting frictions in a model that includes endogenous general equilibrium determination of prices and quantities of debt and equity, time-varying business conditions, and unique stationary equilibria. Our tractable framework may be suitable more generally to study dynamic contracting problems in general equilibrium.

2. Model

This section describes a continuous-time economy in which corporate financial policies may be made in the interests of equity holders. We define the consistency condition which forms our equilibrium concept. We then present solutions at three levels. First, we solve the problem in partial equilibrium where output shocks are the sole source of variation. Second, we extend to the case of time-varying parameter regimes. Finally, we show how to aggregate the economy and solve for the full general equilibrium. To start, we describe the setting, which mostly follows Johnson (2018).

2.1 Firms, Debt, and Discount Rates

Each firm in the economy consists of a single project that produces a non-negative stream of goods. Let $Y^{(i)}$ denote the instantaneous output flow of project i. We assume $Y^{(i)}$ follows the pure-jump stochastic process

$$\frac{dY_t^{(i)}}{Y_t^{(i)}} = \mu \; dt + d\left[\sum_{j=1}^{\mathcal{J}_t}\left(e^{\varphi_j^{(i)}} - 1\right)\right]. \tag{1}$$

Here $\mathcal{J}_t$ is a regular Poisson process with intensity λ, and the percentage jump size is $\varphi_j^{(i)}$. If a jump occurs at time t, the sign of the jump is a Bernoulli random variable (with both outcomes having equal probability). The jump sizes $\varphi^{(i)}$ are drawn from gamma distributions defined over the positive or negative real line, depending on the sign of the jump. A discontinuous cashflow process with random jump sizes is convenient for modeling credit risk because jumping below any given bankruptcy threshold is always a possibility.[7]

Following Admati, DeMarzo, Hellwig, and Pfleiderer (2018) and DeMarzo and He (2018), we assume that firms are allowed to issue or repurchase debt at any time at no cost. Equity finance is also assumed costless. Increases in debt are paid to equity holders; decreases are funded by equity holders. There is no physical capital in the model, and, to keep the focus on financial policy, the firm's stock of (intangible) capital is fixed.[8]

The form of the debt contract is restricted to being a perpetual note. Extending the model to allow finite maturity debt is straightforward. However, the firm faces no choice on maturity, or on any dimension of contract design, and can issue only one class of debt.

Following the usual assumptions in the capital structure literature, we assume the firm receives a tax deduction for coupon interest paid, and that this deduction is realized continuously as long as the firm is alive. When owners of the firm choose to abandon, we assume the project's income stream is permanently reduced by a factor, α, and that creditors inherit the rights to this stream. As usual in this class of models, one non-contractibility built into the set-up is that the firm management cannot commit in advance *not* to act in the interests of equity holders by defaulting when optimal (for equity) to do so. (There are no agency frictions affecting managerial decisions.)

To value the firm's securities, the economy is endowed with a pricing kernel, denoted

[7] The analysis below goes through completely in the case of mixed jump-diffusive dynamics.

[8] The model's qualitative results will not change if we associate a differential unit of physical capital with each firm that must be raised by net external finance at time-zero. Also, as described below, the model can include investment at the household level that changes the aggregate stock of projects.

Λ. Its dynamics (which are to be derived in equilibrium below) are as follows

$$\frac{d\Lambda}{\Lambda} = \eta\ dt + d\left[\sum_{j=1}^{\mathcal{J}_t}\left((u^{-\gamma}-1)1_{\{j,+\}} + (d^{-\gamma}-1)1_{\{j,-\}}\right)\right].$$

Here γ, η, u, and d are constants to be determined in equilibrium. These numbers determine the economy's riskless interest rate as

$$r = -\eta - \tfrac{1}{2}\lambda[(u^{-\gamma}-1) + (d^{-\gamma}-1)].$$

Note that the jump-instance process, $\mathcal{J}$, is the same as the one in the firm's output specification, (1). The model thus assumes jumps are systematic events. Intuitively, $u > 1$ and $d < 1$ correspond to the aggregate fractional change in output on a jump event, and γ corresponds to the relative risk aversion of the representative agent.

2.2 Firm Policies and Asset Values

The firm's financial policy at time-t consists of an amount of debt (the coupon payment per unit time), C_t, and an abandonment decision, i.e., whether not to default at t. Because adjustment is costless, the firm will optimize its policy continuously, given the current state. Because the economy's innovations are independent and identically distributed (*i.i.d.*), the firm's current state is summarized by the level of output Y_t and the inherited level of debt C_{t-}. Due to the pure-jump stochastic structure, nothing happens between output jumps. It follows that default will only occur immediately after a (sufficiently negative) jump, and that, conditional on not defaulting, the firm will continually re-optimize its debt quantity.

At any point in time, the default policy can be characterized by the largest (least negative) output jump size such that default is optimal for any subsequent jump below this level and not otherwise. Let $\underline{\varphi} < 0$ denote this critical value. That is, the default threshold $\underline{Y}$ is $e^{\underline{\varphi}} Y_t$. Because of the costless, continuous optimization (and hence the Markov nature of the policies) and the *i.i.d.* innovations, it is natural to conjecture that $\underline{\varphi}$ itself will be constant.

In fact, we will show that this type of default policies also entail two convenient properties. First, they imply that debt quantity and firm value both scale linearly with output. Second, the policy problem collapses to one dimension: C and $\underline{\varphi}$ are linked by a characterization of optimal default. Because of the first property, we refer to policies in this class as linear. Our analysis will restrict attention to this class.

Linear, homogeneous policies mean that firms are scale-invariant, as in the classic dynamic capital structure model of Goldstein, Ju, and Leland (2001). More recently, in a model of agency conflict, Lambrecht and Myers (2017) show that, under power utility, managers optimally set debt to be linear in the firm's net worth. In a setting similar to ours, without adjustment costs and with downward jumps in asset values, Lambrecht and Tse (2019) analyze the effect of alternate default resolution regimes on a bank whose equity maximizing managers follow a debt policy that is linear in loan value.[9]

We now show that, given a policy in the linear class, we can obtain asset prices in closed-form. We begin with the optimal default condition.

Lemma 1. *Let V denote the value of the firm's project and F the value of its debt. The optimal default policy is for owners to abandon the firm on a jump of V_t below F_{t-}.*

Assume that, prior to default, V and C are linear in output Y and let $v = V/Y$ and $c = C/Y$. Also assume F is linear in C with $p = F/C$. Then the critical default threshold is

$$e^{\underline{\varphi}} = F_{t-}/V_t = cp/v. \tag{2}$$

Note: all proofs appear in Appendix A.

[9] In their setting, the debt is (uninsured) deposits. No commitment issue arises because the market continually adjusts the deposit rate to reflect default risk. There is thus no way for owners to expropriate value from existing creditors.

Besides characterizing the optimal default policy, the lemma is notable in that it provides an alternative and very convenient interpretation of the parameter $\underline{\varphi}$ as simply the log of the market leverage ratio: debt value divided by firm value. We will freely use this interpretation below, and occasionally just refer to $\underline{\varphi}$ as "market leverage."

Next, we use the pricing kernel to compute the value of the firm and the bond, conditional on $\underline{\varphi}$. To economize on notation, define the following quantities

$$\tilde{\mu} = \mu + \tfrac{1}{2}\lambda[u^{-\gamma}(u-1) + d^{-\gamma}(d-1)] \tag{3}$$

$$\tilde{\ell}_d = \tfrac{1}{2}\lambda \, d^{-\gamma} \tag{4}$$

$$\tilde{\ell}_u = \tfrac{1}{2}\lambda \, u^{-\gamma}, \tag{5}$$

and let

$$H(\underline{\varphi}) = \int_{-\infty}^{\underline{\varphi}} e^x \, g^-(|x|) \, dx \tag{6}$$

$$D(\underline{\varphi}) = \int_{\underline{\varphi}}^{0} e^x \, g^-(|x|) \, dx \; + \; \alpha H(\underline{\varphi}) \tag{7}$$

$$U = \int_0^{\infty} e^x \, g^+(x) \, dx. \tag{8}$$

where $g^{\pm}$ stands for the density functions of positive and negative jumps.

Proposition 1. *Given a default boundary, $\underline{\varphi}$, assume the denominators in the following expressions are positive. Then firm value and bond value are linear in output, bond value is linear in the coupon amount, and*

$$p = 1./[r + \tilde{\ell}_d(1 - G^-(|\underline{\varphi}|) - \alpha \, H(\underline{\varphi})e^{-\underline{\varphi}})] \tag{9}$$

$$v = (1-\tau)./[r - \tilde{\mu} - \tilde{\ell}_d \, (D-d) - \tilde{\ell}_u \, (U-u) - \tau e^{\underline{\varphi}}/p] \tag{10}$$

where G^- is the distribution function of negative jumps and α is the cashflow recovery *rate following default.*

2.3 No-commitment Equilibrium

We now are in a position to describe our formulation of the commitment problem. If managers can commit to future leverage policy, then, at time-zero (when the firm originates) owners simply choose $\underline{\varphi}$ to maximize v as given in the proposition above. (Choosing $\underline{\varphi}$ fixes p and v, and therefore c, using the optimal abandonment result.) The result is a version of the Leland (1994) model under costless debt adjustment, with jump dynamics. This case, which we refer to as "full commitment",[10] serves as the benchmark in the subsequent analysis.

Now suppose that, instead, after an initial issuance of some amount $\bar{C} = \bar{c}Y_0$, managers are free to do whatever they want. In particular, they can consider any alternative linear policy $\hat{c}$ (together with the associated default policy). Any such policy, if implemented consistently, makes the value of their equity claim (per unit output)

$$[v - p\hat{c}] \ + \ p[\hat{c} - \bar{c}] = v - p\bar{c},$$

where v and p are determined by the new policy. (The second term is the net proceeds from the purchase/sale of new debt. The first term is the residual equity value post-issuance.) So suppose $\hat{c}$ is chosen to maximize that expression. Then owners immediately face the same choice all over again. Why stick to $\hat{c}$ any more than $\bar{c}$? Given their lack of commitment to *any* policy, there can be no equilibrium unless their proposed policy is self-enforcing. This gives rise to the following definition.

Definition 2.1. *Given the pricing functions v and p determined by Proposition 1, and the default policy $\underline{\varphi}$ implied from Lemma 1 by a given debt amount c, let $\mathcal{C}$ be the set of*

[10] This is slightly inaccurate terminology in the sense that managers still do not commit to forgo inefficient default.

fixed points of the mapping

$$c(\bar{c}) = \arg\max\{v(c) - p(c)\bar{c}\}$$

also satisfying $v - pc > 0$*.*

Then c^* *is a* **linear no-commitment equilibrium** *if and only if* c^* *is the solution to the equity maximization problem:*

$$\max_{c \in \mathcal{C}} \{v(c)Y_t - p(c)C_{t-}, 0\}$$

for all values of the state variables (Y_t, C_{t-})*.*

Belonging to the set of fixed points is a necessary condition for equilibrium, because otherwise there are immediate and continual incentives to deviate. If there are multiple fixed points, then any one of them can only be an equilibrium if there are never incentives for equity holders to switch to another one. As our numerical work will illustrate, we can find unique equilibria for a range of reasonably parameterized and interesting examples, and we present sufficient conditions for this to occur. But it is also possible to construct examples with multiple or zero fixed points.

Implementing a given linear policy involves following a mechanical rule for debt issuance or repurchase. However, following that rule does not mean that managers behave suboptimally. Following a down-jump, for example, manager's problem is, first, whether or not to default, which depends on the level of inherited debt prior to the jump and the cost of implementing the repurchase policy. In addition, if there are alternative linear equilibria, managers could deviate to them. Non fixed-point alternatives are irrelevant. Likewise, at time-zero when the firm has no inherited debt: $\bar{c} = C_{0-}/Y_0 = 0$, implementing a policy $c^\star$ gives equity the payoff per unit output $v(c^\star) - p(c^\star)\bar{c}$ (or $v(c^\star)$). In general, this value will be less than under the alternative policy $\hat{c} = \arg\max\{v(c) - p(c)\bar{c}\} = \arg\max \ v(c)$. However, unless $\hat{c} \in \mathcal{C}$, this value is not attainable, absent commitment. The market will not support the alternative prices.

2.3.1 Equilibrium Properties

What effect does absence of commitment have on firm policies? Intuitively, if a firm issues debt without imposing restrictions on managers, the debt is unprotected against actions that can transfer value from creditors to equity holders. In fact, in a two-period trade-off model it is easy to show that this situation *always* arises: for any non-zero amount of initial debt at $t = 0$, managers will optimally sell more debt at $t = 1$, even if nothing else changes. This is the "ratchet effect" described by Admati, DeMarzo, Hellwig, and Pfleiderer (2018). Equity holders optimally take on more leverage than value maximization would imply. We can readily prove an analogous result (with the aid of some simplifying, but not necessary, conditions).

Corollary 2.1. *Assume that $\alpha = 0$, $r > 0$, and that the probability density function of negative jumps, $g^-(x)$, satisfies $\frac{1}{g^-}\frac{dg^-}{dx} \le -\frac{\tau}{1-\tau}$ and $g^-(0) > \frac{\tau}{1-\tau}\frac{r+\tilde{\ell}_d}{\tilde{\ell}_d}$. Then, with commitment, a unique optimal capital structure exists, characterized by $\underline{\varphi}^C$. If a unique no-commitment equilibrium exists, then it is characterized by higher market leverage: $\underline{\varphi}^{NC} > \underline{\varphi}^C$ and lower firm value.*

The following result provides sufficient conditions (given explicitly in the appendix) for the existence and uniqueness of a no-commitment equilibrium.

Corollary 2.2. *Assume the conditions of Corollary 2.1 hold, and, in addition assume the conditions enumerated in the appendix. Then a unique no-commitment equilibrium exists.*

A necessary condition for existence of a fixed-point in the capital structure problem is that the marginal incentive function $\frac{dv}{dc} - c\frac{dp}{dc}$ (viewed as a function of c) must switch sign.[11] This is different from the settings examined in Admati, DeMarzo, Hellwig, and Pfleiderer (2018), in which the marginal incentives to issue debt are always non-negative. Here it can be the case that, when existing debt is high enough, after a differential reduction $-dc$ the net gain to equity through reduced default risk can be positive. Or, equivalently, the fraction of the firm value increase that accrues to debt holders (which is $c\frac{dp}{dc}/\frac{dv}{dc}$) can be less than one. Referring to equations (10)-(9) and setting $\alpha = 0, \tilde{\ell}_d = 1$ to simplify, it is convenient to differentiate p and v with respect to the default threshold, $\underline{\varphi}$ (and then $\frac{d\underline{\varphi}}{dc} > 0$ appears in both expressions). Then the two terms work out to be

$$c\frac{dp}{d\underline{\varphi}} = -p \; v \; e^{\underline{\varphi}} \; g^{-}(|\underline{\varphi}|)$$

(which uses $c = e^{\underline{\varphi}} v/p$), and

$$\frac{dv}{d\underline{\varphi}} = -v^2 \; e^{\underline{\varphi}} \left(g^{-}(|\underline{\varphi}|) - \frac{\tau}{1-\tau}\frac{1}{p} \right).$$

Cancelling a common $v \; e^{\underline{\varphi}}$, the change in firm value scales like v times the marginal change in default probability, $g^{-}(|\underline{\varphi}|)$, whereas the change in the debt price scales like p times that probability. Typically, $v > p$, and this is the basis for the observation that firm value can increases more than the value gain to existing creditors upon a debt reduction. The remaining term in the second equation $\frac{\tau}{1-\tau}\frac{v}{p}$ is always positive: this is the contribution of an increase dc to tax shields. But the incentive function is negative if

$$(v - p) \; g^{-}(|\underline{\varphi}|) > \frac{\tau}{1-\tau}\,\frac{v}{p}.$$

When c is small, $\underline{\varphi}$ is very negative and the marginal change in default probability is negligible. In this case the tax shield term dominates and the inequality goes the other

[11] This is because the incentive function is zero at any fixed point, and is also a continuous function of c. The incentive is positive for $c = 0$. If there is a zero of the function, then they it become negative for high enough c.

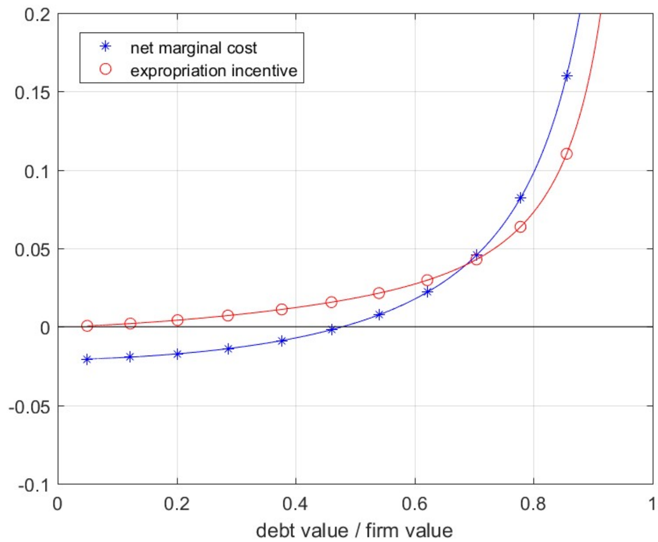
direction. As c increases and $\underline{\varphi}$ rises towards zero, the left side can dominate (depending on the shape of the jump density) leading equity to prefer reduced debt. This may fail for many parameter configurations, and the model will reproduce the ratchet effect. However, for the cases we examine in a which a unique linear equilibrium exists, it *must* be true that equity holders' local incentives always lean in the direction of a debt reduction whenever $c > c^{\star}$, as happens following any down-jump in Y (unless it induces default).

An important question for our subsequent analysis is to understand what factors drive the incentive for extra leverage when commitment is absent. Some helpful intuition emerges from examining the first-order condition for firm-value maximization and comparing it to the analogous condition under equity maximization. The proof of the first corollary above shows that the condition can be written

$$\tilde{\ell}_d\ g^{-}(|\underline{\varphi}|) - \frac{\tau}{1-\tau}\left[r + \tilde{\ell}_d\ (1 - G^{-}(|\underline{\varphi}|))\right] = \tilde{\ell}_d\ g^{-}(|\underline{\varphi}|)\ p/v. \tag{11}$$

Here the left side is just the marginal cost of default losses minus the marginal benefit of tax shields – the standard trade-off terms. Setting these terms equal to zero is the optimality condition under firm value maximization, i.e., with full commitment. The right side is the wedge introduced by the contracting friction. This is the marginal benefit to equity holders of *decreasing* the value of debt. It thus seems accurate to label this an expropriation incentive. Figure 1 shows graphically how this wedge shifts the optimal market leverage (which is $e^{\underline{\varphi}}$) to the right.

Figure 1: First-order conditions



The figure plots the left and right sides of Equation (11). Parameter values are $\sigma = 0.10, \mu = 0.03, \alpha = 0, \lambda = 1, L = 3, \tau = 0.30, \gamma = 4, r = 0.07; d = 0.97, u = 1.03$.

The key intuition in terms of how big the distortion is comes from the simple term $1/v$ on the right side of the first-order condition. This tells us that, other things equal, the wedge is small when firm value is large, and large when firm value is small. Essentially, the value created by optimizing the primary trade-off to capital structure scales up with the value of the firm. But the expropriation incentive does not. Instead, the marginal benefit of adding a unit of leverage – taking the quantity of existing debt as given – only scales with the *unit* value of debt, which is p. So the intuition to bear in mind is that the threat of expropriation is likely to be most significant under adverse conditions for the firm, when the benefits to maximizing marginal enterprise value are low.

2.3.2 Comparison with DeMarzo and He (2018)

Like this paper, DeMarzo and He (2018) (hereafter DH) study the commitment problem in a continuous-time setting with Leland-type debt and no adjustment costs. They restrict attention to debt policies characterized by slow adjustment, $dC = G\ dt$. The first-order condition for equity-maximizing issuance implies that the debt price p must equal $-\frac{\partial V^e}{\partial C}$, where V^e is the value of equity (which is $V - pC$ in our notation). Moreover, if this condition is satisfied, then the issuance policy will always represent a fixed-point in the sense we defined above, because, given any inherited debt level $\bar{C}$, the equity payoff from adjusting to C is $V^e(C) + p(C)(C - \bar{C})$ is maximized at $C = \bar{C}$. A natural question is how their fixed-point policy relates to ours. In particular, if both are valid equilibria, do they lead to economically different conclusions about the impact of commitment on leverage?

As a first clarifying observation, note that the fixed-point argument above does *not* show that discrete debt adjustment policies are suboptimal. Rather, it shows that discretely adjusting one time, *and thereafter* following a DH-type policy is not optimal. However, it does not address *continually* following a discrete adjustment policy, as in our linear class, which leads to completely different pricing functions V and p. The structure of DH's logic is the same as ours: both arguments rule out incentives to deviate to another policy within the class being examined. Neither their analysis nor ours examines deviations to a different class.[12]

In fact, both equilibria can exist and be self-consistent. To confirm this, we generalize the DH model to our case of non-diffusive cash-flow dynamics.[13] In side-by-side comparisons using the same parameter configurations, we confirm that neither the DH policy nor the linear policy dominates the other from the point of view of equity holders.

In terms of economically different implications, a seemingly stark contrast between the DH policies and ours is that in the DH solution, as in the examples in Admati, DeMarzo, Hellwig, and Pfleiderer (2018), debt issuance is always positive, whereas our policies (when an equilibrium exists) feature both issuance and repurchase.[14] However

[12]In that sense, each paper does impose a type of commitment on managers' debt policies.

[13]For technical reasons, the solution fails for pure jump dynamics, so we solve the case with both jumps and Brownian cash-flow shocks. The analogous generalization of our Proposition 1 is also derived. These computations are in an appendix available upon request.

[14]The technical conditions leading to a repurchase incentive are discussed above.

this contrast is less fundamental than it first appears. The DH model does not imply that non-commitment leads to perpetually increasing leverage. Firms can still have "too much" debt (even for equity-maximizing managers) when the cash flow process approaches the default threshold. Although issuance remains positive, it goes to zero rapidly as Y declines. Leverage (C/Y) then falls when the issuance rate drops below the cash-flow growth rate ($\mu > 0$). This results in leverage paths that stay within a stable range.

As a practical matter, the path-dependent nature of the DH model solution would render it too complex for the main purpose of this paper. A key advantage of our linear equilibria is that their simplicity permits us to achieve the goal of extending the analysis of the commitment problem to general equilibrium and time-varying economic conditions.

2.4 Time-Varying Parameters

The model in the previous section presents a stationary capital structure equilibrium in a fully dynamic multiperiod setting. A natural next question is whether the model can be extended to encompass time-variation in the firm's environment? We now show that it can, and generalize the equilibrium concept accordingly.

We consider a two-regime version of the model, in which the regimes are indexed by $s \in 1, 2$. In principle, all the parameters of the model $\sigma, \mu, \lambda, \alpha, L, u, d, r$ are allowed to vary with s. This encompasses variation in either (or both) of the firm-specific, and macroeconomic variables. The state index itself follows a Markov switching process with intensity denoted $\omega(s)$. This process is assumed to be independent of the instantaneous output process Y, implying that Y does not change when a state switch occurs.

Naturally enough, this setting leads us to conjecture that the optimal financial policy and the claims prices remain linear in output, but with coefficients $c(s)$, $v(s)$ and $p(s)$ that depend on the state. It is then straightforward to show that the solution for the optimal default boundary is again given by (2) so that $e^{\underline{\varphi}(s)} = c(s)p(s)/v(s)$.

That threshold determines the firm's response to an adverse output jump. One subtlety that then arises is whether it is *also* ever optimal for owners to abandon upon the event of a switch from one state to the other. Nothing in the set-up precludes this possibility, and modifying the results below to deal with it is straightforward. For simplicity, we will present results for the case in which this is not optimal. This imposes a regularity condition on the model parameters summarized in the following lemma.

Lemma 2. *For a given capital structure policy, and implied claims values, define* **Condition S** *to be the following pair of inequalities:*

$$v(2)/v(1) > \exp(\underline{\varphi}(1)), \qquad v(1)/v(2) > \exp(\underline{\varphi}(2)).$$

If these conditions hold, then it is not optimal for owners to abandon the firm upon a switch between states.

Even though output doesn't change on a state switch, it will in general be true that discount rates change. Specifically, letting $\mathcal{I}_t$ denote the Poisson process counting state switches (whose intensity is $\omega(s)$), the kernel now can be written

$$\frac{d\Lambda}{\Lambda} = \eta(s)\ dt + d\left[\sum_{j=1}^{\mathcal{J}_t} \big((u(s)^{-\gamma} - 1)1_{\{j,+\}} + (d(s)^{-\gamma} - 1)1_{\{j,-\}}\big)\right]$$

$$+\, d\left[\sum_{i=1}^{\mathcal{I}_t} \big((\xi(1) - 1)1_{\{i,1\}} + (\xi(2) - 1)1_{\{i,2\}}\big)\right]. \tag{12}$$

where $1_{\{i,2\}}$ indicates a switch from state 2 to state 1 , and $\xi(2) > 0$ is the ratio of marginal utility in state 1 to that of 2 (and vice versa for $1_{\{i,1\}}$ and $\xi(1)$). If $\xi(1) = \xi(2) = 1$, the regime switches can be viewed as idiosyncratic, or unpriced, events.

Now, define the following risk-adjusted intensities

$$\tilde{o}(s) = \omega(s)\ \xi(s)$$

and, re-define the quantities in (3)-(8) to be their state-dependent counterparts (i.e. using he distribution and densities, $G^{\pm}(s)$ and $g^{\pm}(s)$, of jumps in each state), We then have the following generalization of Proposition 1.

Proposition 2. *Given a pair of values $\underline{\varphi}(s)$, define the following coefficient matrices*

$$K_p = \begin{bmatrix} r(1)+\tilde{o}(1)+\tilde{\ell}(1)(1-G^-(|\underline{\varphi}(1)|)-\alpha(1)H(1)e^{-\underline{\varphi}(1)}) & -\tilde{o}(1) \\ -\tilde{o}(2) & r(2)+\tilde{o}(2)+\tilde{\ell}(2)(1-G^-(|\underline{\varphi}(2)|)-\alpha(2)H(2)e^{-\underline{\varphi}(2)}) \end{bmatrix}$$

$$K_v = \begin{bmatrix} r(1)-\tilde{\mu}(1)+\tilde{o}(1)-\tilde{\ell}_d(1)(D(1)-d(1))-\tilde{\ell}_u(1)(U(1)-u(1))-\tau e^{\underline{\varphi}(1)}/p(1) & -\tilde{o}(1) \\ -\tilde{o}(2) & r(2)-\tilde{\mu}(2)+\tilde{o}(2)-\tilde{\ell}_d(2)(D(2)-d(2))-\tilde{\ell}_u(2)(U(2)-u(2))-\tau e^{\underline{\varphi}(2)}/p(2) \end{bmatrix}$$

Assume the preceding matrices are both positive definite. Then firm value and bond value are linear in output, bond value is linear in the coupon amount, and the bond value p and firm value v are the solutions to

$$K_p p = 1_2 \tag{13}$$

$$K_v v = (1-\tau)1_2 \tag{14}$$

where $1_2 = [1,1]'$.

Under full commitment, a firm that starts life in state s then chooses the pair $\underline{\varphi}$ to maximize $v(s)$.[15] Without commitment, we again require an equilibrium policy to satisfy a consistency condition.

Definition 2.2. *Let $\mathcal{C}$ be the set of policy vectors $\underline{\varphi}$ together with associated price vectors v and p determined by Proposition 2, and coupon vector c determined by the default condition in Lemma 1, such that, in each state, conditional upon the policy in the*

[15]Note that there are, in general, two full-commitment solutions for otherwise identical firms, depending on their birth state, because the same pair of default thresholds will not maximize both $v(H)$ and $v(L)$.

other state, the policy is a fixed point of the mapping

$$c(\bar{c}; s) = \arg\max v(c; c; s) - p(s)\bar{c},$$

and, in both states, and $v(c) - p(c)c > 0$*.*

Then c^* *is a* **linear no-commitment equilibrium** *if and only if* c^* *is the solution to the equity maximization problem:*

$$\max_{c \in \mathcal{C}} \{v(c)Y_t - p(c)C_{t-}, 0\}$$

for all values of the state variables (s_t, Y_t, C_{t-})*.*

Intuitively, unless a policy has this property, then managers have an incentive to deviate instantaneously.

Constructively, one can identify such a joint fixed point (if it exists) by building two loci in the plane defined by $\underline{\varphi}(1) \times \underline{\varphi}(2)$ as follows. First, for each value of $\underline{\varphi}(1)$, find the fixed point $\widehat{\underline{\varphi}}(2)$ of the state-2 equation. This defines a locus $(\underline{\varphi}(1), \widehat{\underline{\varphi}}(2))$. Then, for each value of $\underline{\varphi}(2)$, find the fixed point $\widehat{\underline{\varphi}}(1)$ of the state-1 equation. This defines a locus $(\widehat{\underline{\varphi}}(1), \underline{\varphi}(2))$. If an intersection of the two loci exists, it satisfies the definition above. We will show that this construction works and yields a unique solution in numerical examples in Section 3.

2.5 General Equilibrium

The models developed above will allow us to analyze in the sections below how the firm's parameters and its environment map into differing effects of commitment on policies and claim values. It is also of interest to ask how commitment may affect the economy as a whole. This section explains how the model can be aggregated to compute the implied general equilibrium.

The aggregation is made feasible when it is not necessary to keep track of the distribution of firm characteristics. Hence we assume that the economy is endowed with a continuum of productive project-firms, whose measure is denoted M, all of which are stochastically identical.

Specifically, the output jump counting process, $\mathcal{J}_t$, and the sign of the jump, is assumed common across firms. Thus a jump is a systematic event. However, conditional on the sign, the individual jump incidences are *i.i.d.* across firms. Intuitively, firms differ in their exposure to a systematic event, although the degree is only revealed *ex post*.

Ignoring entry and exit for the moment, we can then integrate over firms to obtain the dynamics of aggregate output Y in state s as

$$dY_t = \mu(s)\ Y_t\ dt + d\ \int_i^M\ Y_t^{(i)} \left[\sum_{j=1}^{\mathcal{J}_t} \left(e^{\varphi_j^{(i)}} - 1\right)\right]\ di$$

$$= \mu(s)\ Y_t\ dt + Y_t\ d\left[\sum_{j=1}^{\mathcal{J}_t} \left(\mathrm{E_t}\left[e^{\varphi_j^{(i)}} | \varphi_j > 0; s\right] 1_{\{j,+\}} + \mathrm{E_t}\left[e^{\varphi_j^{(i)}} | \varphi_j < 0; s\right] 1_{\{j,-\}} - 1\right)\right]. \tag{15}$$

where $1_{\{j,+\}}$ and $1_{\{j,-\}}$ are indicators for the sign of the jth jump. Applying a law of large numbers, the stochastic term is

$$Y_t\ d\left[\sum_{j=1}^{\mathcal{J}_t} \left(\Phi^+(t)1_{\{+\}} + \Phi^-(t)1_{\{-\}} - 1\right)\right]$$

where $\Phi^{\pm}(s)$ are the exponential integrals over the positive and negative jump size distributions. Thus aggregate output follows a binomial process. Conditional on the state and the sign of the jump, the size of aggregate shocks is not random. For up-jumps, we conclude that the aggregate parameter $u(s)$ is equal to $\Phi^+(s)$.

On a down-jump event, we have already shown that it will be optimal for owners of firms to default if they experience a drop of $\log(Y_t^{(i)}/Y_{t-}^{(i)})$ below $\underline{\varphi}(s)$. We have assumed that for all such firms output is thereafter reduced by the factor $\alpha(s)$, which may be zero. The effect of exit on output is simply to alter the downward aggregate jump size. In equation (15) above, we replace the down-jump expectation $\mathrm{E_t}\left[e^{\varphi_j^{(i)}}|\varphi_j^{(i)} < 0; s\right]$ with

$$\mathrm{E_t}\left[e^{\varphi_j^{(i)}}|\underline{\varphi} < \varphi_j^{(i)} < 0; s\right] + \alpha \mathrm{E_t}\left[e^{\varphi_j^{(i)}}|\varphi_j^{(i)} \leq \underline{\varphi}; s\right].$$

This is exactly the function defined above as $D(\underline{\varphi}, s)$. Aggregation yields the condition $d = D$. (And also $U = u$.) General equilibrium thus links the firms' optimal policies to the aggregate risk.

The results below will continue to assume that it is not optimal for firms to exit upon a switch of state in either direction. (Hence the sufficient conditions in Lemma 2 need to be verified in any given solution.) The assumption is sensible in general equilibrium since the alternative is the simultaneous default of all firms in the economy.

Turning to entry, we assume that households use their resources to create a flow of new projects, which increases the mass dM/M at an exogenous rate. The flow of new projects shows up as an additional term in the growth rate, μ, of aggregate output, dY/Y.

When new projects are created, they are distributed uniformly across households. Each household sells its projects to all the others. Each firm then sells its initially optimal quantity of debt, the value of which passes to the equity holders. These financial transactions between households and themselves result in no net flow of real goods. Households' aggregate income is assumed equal to Y_t.[16]

Next, we assume there is a representative household characterized by preferences of the stochastic differential utility class (Duffie and Epstein (1992), Duffie and Skiadas (1994)), the continuous-time analog of Epstein and Zin (1989) preferences. Specifically,

[16] A government sector is assumed to collect corporate taxes, net of tax shields, and rebate any surplus to households.

agents maximize the lifetime value of the consumption stream C, defined as

$$J_t = \mathrm{E_t}\left[\int_t^{\infty} f(C_u, J_u)\ du\right].$$

where

$$f(C, J) = \frac{\beta C^{\rho}/\rho}{((1-\gamma)\ J)^{1/\theta-1}} - \beta\theta J.$$

Here β is the rate of time preference, γ is the coefficient of relative risk aversion, $\rho = 1 - 1/\psi$, where ψ is the elasticity of intertemporal substitution, and $\theta \equiv \frac{1-\gamma}{\rho}$. (We assume $\gamma \neq 1, \rho \neq 0$.) Absent investment, $C = Y$. Since Y's dynamics have already been determined, it is straightforward to solve for the value function, which determines marginal utility, and hence the pricing kernel and the interest rate.

Proposition 3. *The household's value function is $J = j(s)\ Y^{1-\gamma}/(1-\gamma)$, where $j(s)$ is the solutions to two coupled algebraic equations given in the appendix. The pricing kernel takes the form given in (12) with*

$$\eta(s) = \beta\theta\ [(1 - \frac{1}{\theta})\ j(s)^{-\frac{1}{\theta}} - 1] - \gamma\mu$$

$$\xi(1) = (j(2)/j(1))^{1-\frac{1}{\theta}}, \qquad \xi(2) = (j(1)/j(2))^{1-\frac{1}{\theta}}.$$

The riskless interest rate is then given by

$$r(s) = -(\eta(s) + \tfrac{1}{2}\lambda(s)((d(s)^{-\gamma} - 1) + (u(s)^{-\gamma} - 1)) + \omega(s)(\xi(s) - 1)).$$

The results above show how optimal firm policies feed back into the equilibrium quantities d, u, ξ, and r. These parameters, in turn, are sufficient to characterize the macroeconomy from the perspective of the firm's problem. The natural solution procedure is therefore to simply iterate: starting from initial values of d, u, ξ, and r, obtain the optimal leverage $\underline{\varphi}$, then update d, u, ξ, and r, and repeat. In the numerical examples below, the procedure converges rapidly to a unique solution as long as the starting parameters obey the regularity conditions given in the propositions.

3. The Private Value of Commitment

We now turn to the examination of the model's implied properties. In particular, this section focuses on the cost of lack of commitment to the firm. We illustrate the characteristics that lead to greater or lesser cost, both in the cross-section and in the time-series. The analysis in this section is set in partial equilibrium; we consider general equilibrium implications in Section 4.

3.1 Cross-Sectional Implications

To start, we numerically illustrate the difference between committed and non-committed capital structure for a range of firm parameters for the single-regime case where there is no time-variation in parameters. The exercise holds the economy-wide parameters (u, d, r, γ, τ) fixed. Also, for simplicity, all numerical examples throughout the paper will assume that intensity of output jumps is $\lambda = 1$, i.e., roughly one jump is expected per year.

Recall the model's assumption is that the logarithm of the downward jumps in output are drawn from a gamma distribution with mean of σ and standard deviation of $L\sigma$. The distribution of the upward jumps in output plays no role in determining capital structure or solving the model. For numerical analysis, we will assume that the (log) upward jumps are drawn from an exponential distribution satisfying the property that $\mathrm{E}[Y'/Y|\mathrm{up}] = 1/\mathrm{E}[Y'/Y|\mathrm{down}]$. In this sense, the jump distribution is symmetrical.[17] With these assumptions, the annual standard deviation of log output changes is roughly

[17]The right-hand side of this expression turns out to be $(1 + \sigma L^2)^{1/L^2} \equiv 1/D$. To satisfy our distributional assumption, the corresponding exponential parameter for the up-jumps is $1 - D$.

2.5 times σ when $L = 3$. This also the volatility of asset value in this version of the model, since asset value is proportional to output.

Our analysis in Section 2 showed that the no-commitment equilibrium produces more leverage than firm value maximization. Table 1 shows the magnitude of the distortion in debt policy for a range of firm characteristics. Columns with and without commitment are labeled C and NC, respectively. The first two columns report the quantity of debt scaled by output(or "book leverage"). The middle columns report the market value of debt scaled by the market value of assets. The last two columns show the credit spread on the debt. The first row shows results for a baseline set of firm parameters. Each of these parameters is then varied to a higher and lower value in the three subsequent pairs of rows.

From the first row, we see that firms without commitment take on about 50 percent more debt than those with commitment. This excess debt increases the default risk and more than doubles the credit spread on the firm's debt. Because of the lower price (higher yield), the increase in debt looks less dramatic in market value terms, but still results in a leverage increase of 22 percent of the unlevered asset value.

From the two middle rows, we see that in terms of market leverage, the distortion is much higher when firm growth is low, but is not much changed with the level of output volatility or of the recovery rate. While the quantity of excess debt increases with σ and α, the lower price of the debt essentially off-sets the increase.

Table 2 shows the valuation consequences of the excess leverage under no-commitment. The first two columns report the firm valuation multiplier; the next column gives the discount expressed as a multiple of income; the right-most column expresses the discount as a percentage of the value under commitment. The firm parameter values in each row are the same as in the preceding table.

Two main observations from this Table are (1) that the loss of investor surplus can be economically large; and (2) these theoretical values are plausible relative to empirical

Table 1: Capital Structure : Committed *vs* Non-Committed

	book leverage (C/Y)		market leverage (F/V)		credit spread ($y-r$)	
	C	NC	C	NC	C	NC
$\mu = 0.03$, $\sigma = 0.10$, $\alpha = 0.25$	0.95	1.52	0.55	0.77	2.30	4.82
$\mu = 0.00$	0.53	1.18	0.55	0.93	2.30	9.37
$\mu = 0.06$	4.52	4.96	0.55	0.59	2.30	2.65
$\sigma = 0.05$	1.03	1.42	0.62	0.78	1.11	2.61
$\sigma = 0.15$	1.01	1.65	0.54	0.76	3.50	6.38
$\alpha = 0.0$	0.77	1.28	0.48	0.69	2.06	4.38
$\alpha = 0.5$	1.27	1.85	0.67	0.85	2.73	5.24

The table reports theoretical firm properties under capital structure commitment (columns labelled C) and no-commitment (NC) using the model of Section 2.2. The baseline firm parameters are shown in the top row left-hand column. Other values are $L = 3, \lambda = 1, \tau = 0.3, r = 0.07, d = 0.97, u = 1.03, \gamma = 4$.

estimates. Matvos (2013) estimates the value of an array of bond covenants in a sample of syndicated loans. In terms of pricing, he finds that including two covenants (the median in his sample) reduces the credit spread on the loan by 50 percent, on average, which accords with our findings in Table 1. His methodology also produces estimates of the issuing firm's surplus from including protective covenants, relative to the alternative of issuing unprotected debt. On average, the surplus is 52 percent of the credit spread. This can be compared with our valuation units in Table 2 by capitalizing the perpetuity value of the credit spread. Using the parameters in the first row, protected debt carries an interest rate of 9.3 percent and the riskless rate is 7.0 percent. So a risky unit perpetuity has a price discount of 3.53 ($1/.07 - 1/.093$). The firm has a debt quantity of 0.95 times income. So the price discount is 3.36 times income. One-half this amount (Matvos's estimate) is 1.68 times income, which is almost precisely the surplus value shown in the table (1.67).

Recently, Green (2018) has estimated the valuation of the full covenant packages found in high-yield bonds. His point estimate is 2.4% of firm value, which, while economically large, is smaller than most of the values in the right-hand column of Table 2. His model (and structural estimation) allow covenants to have real costs, however, which in our setting they do not.[18] While he interprets the cost in terms of inefficient restrictions (e.g. on asset sales or investments) due to limitations in available covenants, they could also include direct costs of monitoring and enforcement. Green's estimates are thus net benefits, whereas the numbers in the table are gross.

Table 2: Cost of Unprotected Debt

	firm value (V/Y)		value loss	
	C	NC	difference	percent
$\mu = 0.03$, $\sigma = 0.10$, $\alpha = 0.25$	18.42	16.75	1.67	10.00
$\mu = 0.00$	10.30	7.75	2.55	32.82
$\mu = 0.06$	87.58	86.64	0.94	1.09
$\sigma = 0.05$	20.60	19.12	1.48	7.76
$\sigma = 0.15$	17.76	16.21	1.55	9.57
$\alpha = 0.0$	17.79	16.39	1.40	8.52
$\alpha = 0.5$	19.62	17.75	1.87	10.51

The table reports theoretical firm properties under capital structure commitment (columns labelled C) and no-commitment (NC) using the model of Section 2.2. The baseline firm parameters are shown in the top row left-hand column. Other values are $L = 3, \lambda = 1, \tau = 0.3, r = 0.07, d = 0.97, u = 1.03, \gamma = 4$.

In terms of cross-sectional variation, the table also highlights firm characteristics that make commitment relatively more or less valuable. Most prominently, the model implies very little surplus from covenants for high-growth firms, and very large surplus for low-growth firms. The model does not embed a depiction of the contracting technology – and its cost function – that could lead some firms to optimally pay to achieve protected debt, while others choose not to do so. However, unless that contracting cost itself is a

[18]Indeed, Green estimates that a full array of high-yield covenants would have a (small) net negative impact on the value of an investment-grade firm.

steeply declining function of firm profit growth, the model does offer the prediction that covenant usage should be rarer in high-growth firms and more frequent in low-growth ones.

This prediction also finds some empirical support. See Nash, Netter, and Poulsen (2003), Demiroglu and James (2010), and Reisel (2014).[19] The rationale sometimes offered in the literature is that covenants may be unsuitable for high-growth firms because they impose limits on flexibility and may impede growth opportunities. Interestingly, our model's result has nothing to do with flexibility or investment. Instead, as described in Section 2, the expropriation incentive that leads to excess leverage scales inversely with firm value. Intuitively, managers of very valuable (high-growth) firms have much more incentive to get the first-order trade-off between tax-shields and default costs right, and less incentive to worry about transfers from creditors.[20]

The results here also allows us to deduce the model's implications about what other firm characteristics are likely to be associated with covenant use. In particular, it speaks to the (naive) conjecture that firms with unprotected debt would be expected to have higher leverage. The model implies that covenant usage is mostly determined by expected growth rate. Hence, we could reasonably approximate the cross-section as containing low-growth covenant-using firms (i.e., with commitment) and high-growth firms who forego covenants (no commitment) Referring to the second and third rows of Table 1, we see that the first group would optimally choose market leverage ratio 0.55, while the second group would optimally choose 0.59. The precise numbers are, of course, parameter specific. The point is that they are quite similar: the model does not predict that unprotected debt must necessarily be strongly associated with higher leverage.[21]

[19]Billett, King, and Mauer (2007) and Bradley and Roberts (2015) report evidence that covenant use increases with growth opportunities, however.

[20]This intuition is formalized in the discussion in Section **??**.

[21]That said, it does turn out that the naive prediction is true in our empirical work in Section 5.

3.2 Dynamic Implications

We now turn to the dynamic model developed in Section 2.3 to understand the model's implications for time-series variation in the capital structure distortion due to uncommitted debt policy. We consider two types of applications. First, we consider (idiosyncratic) variation in the firm's parameters, holding the economy-wide parameters fixed. We next consider simultaneous variation in firm and macroeconomic conditions (as, for, example in Hackbarth, Miao, and Morellec (2006)). All the cases will adopt the assumption that there is a "good" state that is unconditionally more likely, with a half-life of 10 years, and a short-lived "bad" state, with a half-life of 10 quarters.

A first finding is that the model implies more countercyclical leverage for firms without commitment. The basic logic of the trade-off setting implies that when risk increases or growth decreases, increasing potential default costs lead to less leverage. In Table 3, we quantify this via the ratio of debt (or leverage) in the bad state to that in the good state. The first column tells us that, for these parameter values, firms with commitment lower their debt quantities in bad times by multiples from 0.48 to 0.87. The third column indicates similar contractions in market leverage, except in one case (the first row) where lower debt leads to bond prices that are high enough that the market leverage increases. These policies contrast with the no-commitment cases in the second and fourth columns. There we see debt quantities and leverage that can sometimes *increase* in bad times. In the cases where there is still a decrease, that decrease is markedly smaller than in the corresponding case with commitment. Again, the intuition behind this result is that expropriation incentives decrease when the firm value is higher.

Given that capital structure is relatively more distorted without commitment in bad times, it is not surprising that the surplus loss to firms is also higher in those times. This is shown in Table 4, where the percentage loss of firm value in the no-commitment cases is reported in each state. The losses are only mildly larger in the bad state in the

Table 3: Cyclicality of Capital Structure Policy

	debt quantity bad-state/good-state			
	book leverage		market leverage	
	C	NC	C	NC
switching variables:				
$\mu^{(i)}$	0.8689	1.0060	1.0161	1.1559
$\sigma^{(i)}$	0.7470	1.0411	0.7483	0.9974
$\mu^{(i)}, \sigma^{(i)}$	0.7077	0.8788	0.8171	0.9891
r, d, u, ξ, and				
$\mu^{(i)}$	0.6402	0.9046	0.8590	1.1217
$\sigma^{(i)}$	0.6110	0.8983	0.6557	0.9096
$\mu^{(i)}, \sigma^{(i)}$	0.4827	0.8863	0.6531	1.0563

The table compares the optimal debt policies for the 2-regime model with and without capital structure commitment. The numbers reported are the ratios of debt amounts in the bad state to the amount in the good state. The first two columns measure debt as coupon expense as a fraction of output (C/Y in the model). The second two columns measure debt as market value of debt as a fraction of firm value (F/V). In the first three rows, the aggregate state does not switch, and is described by the parameters $r = 0.06, d = 0.95, \xi = 1$. The firm-specific growth rate and volatility switch, and take on the [good-state, bad-state] values values $\mu = [0.06, 0.00]$ and $\sigma = [0.05, 0.15]$. In the bottom three rows the aggregate state also switches, and is characterized by the pairs $r = [0.07, 0.01], d = [.95, .88], \xi = [2.0, 0.5]$. All cases use $u = 1/d, \alpha = 0.25, L = 3, \lambda = 1, \gamma = 4$, and $\omega = [0.07, 0.28]$.

first three rows where the states are idiosyncratic. However the next three rows, where macroeconomic parameters also vary, show a notably larger surplus loss in bad times.

This finding also finds support in the empirical literature. Countercyclical covenant use is documented by Bradley and Roberts (2015) and Helwege, Huang, and Wang (2017). Likewise, procyclical issuance of "cov-lite" debt is noted by Becker and Ivashina (2016) among others. The fact that investors apparently exhibit an increasing appetite for unprotected debt in expansions is sometimes viewed as evidence of irrational exuberance or desperation for yield. Our model offers a different perspective. Since the relative benefits of covenants are smaller in good times, it follows that we should expect fewer firms to use them.[22]

[22] We note again the qualifier that we have no explicit model of variation in contracting costs across

Table 4: Cyclical Cost of Unprotected Debt

	percent reduction in firm value without commitment	
	good state	bad state
switching variables:		
$\mu^{(i)}$	4.44	5.26
$\sigma^{(i)}$	9.67	11.29
$\mu^{(i)}, \sigma^{(i)}$	2.46	3.24
r, d, u, ξ, and $\mu^{(i)}$	17.85	21.98
$\sigma^{(i)}$	15.74	18.82
$\mu^{(i)}, \sigma^{(i)}$	15.86	21.40

The table shows the percentage decrease in firm value without commitment to debt policy for the 2-regime model. In the first three rows, the aggregate state does not switch, and is described by the parameters $r = 0.06, d = 0.95, \xi = 1$. The firm-specific growth rate and volatility switch, and take on the [good-state, bad-state] values values $\mu = [0.06, 0.00]$ and $\sigma = [0.05, 0.15]$. In the bottom three rows the aggregate state also switches, and is characterized by the pairs $r = [0.07, 0.01], d = [.95, .88], \xi = [2.0, 0.5]$. All cases use $u = 1/d, \alpha = 0.25, L = 3, \lambda = 1, \gamma = 4$, and $\omega = [0.07, 0.28]$.

4. The Social Cost of Noncontractability

Our model has described how firms with unprotected debt (without commitment) adopt financial policies that feature more – and more countercyclical – leverage. An implication of this is that such policies lead to higher default rates and real losses in bad states of the world, compared to a world where managers are constrained to act in the interests of the entire firm. In general equilibrium, these increases in defaults in bad states lead to higher marginal utility, which feeds back (via the pricing kernel) to managers' policy decisions, which reflect the exposure of debt claims to systematic risk.

In this section, we analyze these general equilibrium implications. The equilibrium version of the model takes into account the above feedback mechanism, as well as effects that operate through the riskless interest rate.[23] The theory permits a precise quantifi-

firms or economic states. If costs of writing and monitoring covenants also go up in bad times, then the model's prediction about their adoption could be ambiguous.

[23]In the general version of the model with household savings, another channel affecting welfare is the feedback from the cost of capital to investment, which affects the growth rate of output. When the

cation of welfare effects through its closed-form expression for the value function of the representative household, as described in Section 2.4.

The model economy is, of course, quite stylized and omits many important channels through which financial policies could have real effects. For this reason, we do not undertake a structural estimation or detailed calibration of the parameters. Our interest is in the incremental welfare cost of moving from an economy with commitment to one without commitment. We have already computed (in partial equilibrium) the losses that accrue to individual firms from noncontractability. Here we extend the analysis to the entire economy, taking into account the externality that excessive debt imposes on aggregate risk. We present the model's quantification of these welfare effects and show how they vary with the parameter assumptions.

Table 5 contrasts the macroeconomic outcomes of two otherwise equal economies, one having committed debt policies (in the columns labeled C) and one without commitment (columns NC). The computation uses the 2-regime version of the model in which the first and second moment of output switch across states. (The parameters are given in the table caption.) The transition probabilities imply that the economies have "good" and "bad" states with half-lives of, respectively, 10 years and 2.5 years. Results are shown for three levels of risk aversion.

The first two columns quantify the increase in unconditional aggregate risk when there is no commitment. The excess leverage that firms take on results in an increasing level of default losses, which raises the average output volatility on the order on one percent. Since agents are risk averse, this excess volatility is a primary channel driving welfare costs. Households' welfare, as captured by the value function, quantifies lifetime expectations, and hence also depends on intertemporal discount rates. The second and third pair of columns exhibit the effect of noncontractability on two components of

elasticity of substitution exceeds one, increased risk lowers investment. Hence the level and cyclicality effects of lack of commitment harm welfare more than in the cases presented here.

discount rates. In columns (3)-(4), the riskless interest rate is seen to decline without commitment. This is due to increased precautionary savings incentives caused by higher risk. Columns (5)-(6) show that the required compensation for bearing this risk – the risk premium – also rises without commitment. Finally, the right-most pair of columns measures the cyclicality of the equilibria via the ratio of marginal utility in the bad state to that of the good state. This number can be interpreted as "how bad' the bad times are. The answer is that they get worse without commitment. When $\gamma = 6$ (a moderate value in the asset pricing literature), for example, bad times are almost 60 percent worse without commitment.

Table 5: Committed *vs* Non-Committed Debt : Economy Properties

	output volatility		interest rate		risk premium		cyclicality	
	C	NC	C	NC	C	NC	C	NC
$\gamma = 2$	3.65	4.43	7.67	7.13	0.36	0.54	1.16	1.21
$\gamma = 4$	3.58	4.47	7.42	6.57	0.82	1.34	1.53	1.77
$\gamma = 6$	3.51	4.93	7.15	5.05	1.33	3.19	2.09	3.33

The table shows unconditional moments in 2-state general equilibrium economies with committed (C) and non-committed (NC) financial policy. Numbers in the first six columns are annual percentages. The economies are characterized by switching first and second moments of output growth whose values in the two regimes are $\mu = [0.04, 0.00]$, and $\sigma = [0.025, 0.075]$. The regime switching intensities are $\omega = [.07, .28]$ implying a half life of duration 10 years in state 1 and 2.5 years in state 2. The other parameters are $\tau = 0.3, \lambda = 1, L = 3, \alpha = 0, \beta = 0.06, \psi = 1.5$.

Table 6 shows how these properties translate into social losses for a range of preference assumptions. The left-hand column of the table reports the private cost to noncontractability in each of these economies. Here, the computation reports the decline in the value of firms as a fraction of income, $v_{NC}/v_C - 1$, when comparing economies with and without commitment.[24] In line with the results in Section 3, this loss of private surplus

[24]For each comparison in the table, we take the unconditional average of the reported quantity across states s, for both economies. All values in the table are negative.

is economically very substantial. The calculations in Section 3 held the macroeconomic parameters constant, whereas here the comparison reflects the differing pricing kernels in the two economies.

The next column in the table computes the social losses when a single firm loses the ability to commit. Here we fix the economic parameters at their full-commitment values, and compute the percentage loss in a claim to firm output $Y^{(i)}$ (i.e., without taxes or tax shields) due to the added default risk.[25] The losses computed this way are similar in magnitude to the first column

The main result in the table is the third column that computes the full social cost in general equilibrium. According to Proposition 3, the form of households' value function is $j(s)\ Y^{1-\gamma}/(1-\gamma)$. So the change in value between two economies can be expressed in terms of equivalent fractions of permanent income via the change in the certainty equivalent $j(s)^{\frac{1}{1-\gamma}}$. For all values of the preference parameters, these losses are extremely large. Taken at face value, the results imply that the cost of noncontractability to these economies is on the order of 20 percent of income.

The crucial additional insight from this computation is that the general-equilibrium loss social losses are much larger than the private losses or the partial equilibrium losses in the first two columns. (Note that the columns are all expressed in comparable units, i.e., as losses of value as a fraction of income.) This conclusion, which is not strongly sensitive to the preference parameters, indicates that social incentives to restrict excess leverage exceed private incentives. Hence that there is a potential policy motivation to enhance capital structure contractability.

We have emphasized two distortions due to lack of commitment: more default, and more cyclical default. The right-most columns in the table decompose the welfare cost into components due to each channel. In the column labeled "debt level" we compute the value function for an economy with the same degree of cyclicality as in the full-

[25] For the case on the sixth line, a no-commitment equilibrium does not exist.

commitment economy, but having the same unconditional value of market leverage as the no-commitment economy. The numbers verify that raising the debt level is responsible for the largest part of the welfare losses. However the column labeled "cyclicality" still shows an economically large residual contribution from the leverage dynamics. Indeed, for higher levels of risk aversion, the two components are of the same magnitude.

Table 6: Costs of Non-Commitment

	PRIVATE COST	SOCIAL COST			
				due to:	
		PEQ	GEQ	debt level	cyclicality
$\gamma = 2$	7.57	8.86	12.54	11.44	1.10
$\gamma = 4$	11.87	10.00	18.23	13.43	4.80
$\gamma = 6$	19.60	10.94	36.74	20.81	16.73
$\psi = 1.0$	12.70	11.81	17.70	13.20	4.40
$\psi = 2.0$	11.21	8.96	18.47	13.59	4.88
$\beta = 0.04$	8.46	–	14.16	10.22	3.94
$\beta = 0.08$	14.08	14.45	20.74	15.87	4.87

The table reports the percentage losses due to non-contractability in financial policy in economies with time-varying growth and uncertainty. Each comparison reports changes in unconditional average across regimes. The private cost is the loss in value to owners of firms, expressed as a percentage of firm output. The social cost in partial equilibrium (PEQ) is the loss in value of a claim to firm output when a single firm cannot commit to its capital structure. The social cost in partial equilibrium (GEQ) is the percentage reduction in the representative agent's value function between economies with and without commitment, expressed in terms of equivalent loss in aggregate output. The fourth column computes the welfare loss in an economy that has the same average level of debt as the non-commitment case, but the cyclicality of the full-commitment case. The fifth column is the difference between the third and fourth columns. In the first three rows $\beta = 0.06, \psi = 1.5$. The middle two rows use $\beta = 0.06, \gamma = 4$. The bottom two rows use $\psi = 1.5, \gamma = 4$. The economies are characterized by switching first and second moments of output growth whose values in the two regimes are $\mu = [0.04, 0.00]$, and $\sigma = [0.025, 0.075]$. The regime switching intensities are $\omega = [.07, .28]$ implying a half-life of duration 10 years in state 1 and 2.5 years in state 2. The other parameters are $\tau = 0.3, \lambda = 1, L = 3, \alpha = 0$.

Of course, there are numerous caveats to the magnitudes reported here. The computation assumes full loss of output from defaulted projects. Raising the recovery parameter to, e.g., $\alpha = 0.50$ substantially reduces both private and social losses. However, our conclusion regarding the degree of externality (i.e., the relative size of the two costs) is not affected. The model does not account for any costs of achieving commitment, for example from monitoring and enforcement. These burdens would lower the private cost of noncommitment, but leave the social cost unaffected. Also, the model includes no special social *benefits* to debt, e.g., through resolving asymmetric information or moral hazard problems, and thus potentially increasing total investment. However, again, the comparisons we are drawing is between economies with and without commitment, where both include the same debt contract. It seems harder to argue that any social benefit to debt would *increase* when policy commitment is removed as a possibility. It is true that the analysis here compares polar extremes: full commitment versus none. In real life, both protected and unprotected debt exist. Having noted this, it is also true that the analysis here considers only one dimension of contractability, namely, the firm's leverage. In the real world, financial policy is multidimensional. Even debt policy encompasses numerous dimensions (including maturity, seniority, collateral, etc.) along which managerial choice could diverge from firm value maximization. The degree to which our estimates overstate losses that are due to partially incomplete contracting along many dimensions thus requires further analysis.

5. Empirical Evidence

A key prediction from the model is that, in the absence of commitment to capital structure, firms will exhibit a greater degree of countercyclicality of leverage policy. That is, they will tend to increase leverage more in bad times (or decrease it less) than they would under complete contracting. This countercyclicality contributes an extra component to the general equilibrium effects documented in the previous section, on top of the effect of unconditionally higher leverage from lack of commitment. We now bring this prediction to the data.

We test the idea using cross-sectional variation in debt protection. Before examining the results, it is important to first address the endogeneity of that protection. We argued in Section 3 that firms that choose unprotected debt will be the ones for which the surplus loss from lack of commitment to value-maximizing policies is smaller. In the context of the model, we also showed that the largest driver of the surplus differential is firms' expected growth rate, or, more generally its valuation ratio (e.g. Tobin's Q). For the current section to provide valid tests of the model prediction, we need to verify that, when protection choice is driven by the *unconditional* surplus gain from commitment, this does not affect the *conditional* cyclicality differential across firms with and without protection.

Indeed, for a range of numerical examples like those in Section 3, this is the case. For example, contrasting a low-growth firm ($\mu = [0.03, 0.00]$ in the two states) with commitment, to a high-growth one ($\mu = [0.06, 0.03]$) without commitment, the former optimally reduces its outstanding debt (C) by 21% in the bad state while the latter reduces only by 15%, in line with our hypothesis.[26] While this is not to dismiss all concerns regarding endogenous debt protection, it does show that, at least within the theory, the prediction we wish to test is robust to this possibility.

Next, the main empirical challenge is to find a measure for firms' degree of commitment. A firm with outstanding syndicated loans typically has a set of financial covenants that restrict the firm's accounting ratios and financial quantities within a specific range. A firm with stringent covenants therefore has less scope to adjust its financial policies to exploit existing creditors, simply because doing so would risk relinquishing control rights to those creditors. Prior research (e.g., Chava and Roberts (2008)) has demonstrated that the presence of such covenants and the associated risk of transfer of corporate control following covenant violation does, in fact, restrict the borrowing firm's investment,

[26] Other parameters for this calculation are as given in the caption to Table 3.

payout and debt financing policies *ex ante*. In this sense, covenant protection is a natural proxy for the degree of capital structure commitment.

Following the existing literature, we use two measures for covenant strictness of a loan contract based on the LPC-Dealscan database. DealScan reports contract details from syndicated and bilateral loans collected by staff reporters from lead arrangers and SEC filings starting from 1981. The first measure is simply the (log) number of covenants of each loan package.[27]

The second measure, first proposed by Murfin (2012), is the estimated *ex ante* probability of covenant violation when the loan contract is initiated. In practice, there is a wide range of covenant strictness both within and across loan packages. The calculation assumes that the changes in financial ratios follow a multivariate normal distribution with mean zero and a variance-covariance matrix that varies across industries and over time. From this distribution, we then compute the probability that firm's own ratio will fall outside the restricted range during the life of the loan.[28] We apply this estimation to our sample, and extend the sample period to the present. Following Murfin (2012), we exclude loan contracts containing covenants that appear to be violated at the beginning of the contract. However, our results are robust if we assume that the violation probability for those contracts are 1.0.

For each firm-quarter, we calculate the two measures of protection for all the active loan contracts that the firm has, and take the maximum of each as our measures of firm covenant strictness for that particular quarter. The covenant strictness measure requires more non-missing financial observations resulting in a smaller sample size. (In particular, it is not computable for any of the loan observations in the 1980s). We note that our

[27]Loan transactions are reported in both facility-level and package-level in LPC-Dealscan database, where a package is a collection of facilities. We calculate our covenant strictness measures at package level, since covenants are only reported at package level. The tests below restrict the sample to firm-quarters in which there is at least one covenant (so that the log is well defined). But results are not affected by including loans without covenants and using the raw number (i.e., not the log).

[28]For details about the construction of this measure, please refer to Murfin (2012).

measures do not take into account potential covenants in other debt instruments of the sample firms (e.g., public bond issues) that are not covered in Dealscan. The sample also does not include any observations of firms that do not have outstanding syndicated loans. For a detailed description of the selection effects of the LPC-DealScan database, see Chava and Roberts (2008).

Table 7 presents the summary statistics of the two samples in our study, corresponding to the periods in which each of the commitment proxies can be computed. Despite losing roughly 40 percent of the raw covenant observations, the characteristics of the Murfin firm-quarters are very close to those of the larger sample.

We are interested in the relationship between a firm's debt policy and its covenant strictness. We use the quarterly COMPUSTAT database to compute measures of financing activities and other control variables.[29] For the debt financing measure, we use changes in total financial debt ($dlcq + lttq$) scaled by lagged total assets (atq). Welch (2011) points out that the above ratio is problematic in the sense that it treats non-financial liabilities as equity instead of debt. He proposes two alternative denominators for leverage ratios: (i) lagged sum of financial debt and book value of equity ($seqq$); and (ii) the lagged sum of financial debt and market value of equity, defined as the product of common share price and common share outstanding ($prccq \times cshoq$). We use these two alternative scalings as robustness checks.

Other control variables follow respective measures in the literature (e.g., Covas and den Haan (2011)). We control for the following firm-level variables: logarithmic total assets ($\log(atq)$); Tobin's Q is the sum of market value of equity, capital value of preferred stock (pstkq), dividend paid on preferred stock ($dvpq$) and total liabilities (ltq), over total assets; Cash flow is defined as the sum of income before extraordinary items (ibq) and depreciation and amortization (dpq) scaled by lagged total assets; Asset tangibility is

[29]The COMPUSTAT variable codes are given in italics.

defined as the net value of property (*ppentq*), plant and equipment over total assets.[30]

The key prediction of the model is that compared with firms without commitment, firms that can commit to their capital structures reduce their debt more in bad times. Following the recent asset pricing literature, literature (e.g., Joslin, Priebsch, and Singleton (2014); Giglio, Kelly, and Pruitt (2016)), we use two proxies for the aggregate state, based on the Chicago Fed National Activity Index (CFNAI), a weighted average of 85 monthly indicators of US economic activity. The first measure is the composite index of all indicators. Then, as a second measure, we use the sub-index (the Production and Income series) that focuses on the conditions of the corporate sector. For both indexes, negative values indicate below-average growth (in standard deviation units); positive values indicate above-average growth. To facilitate the interpretation of the results, we define our bad-times variable (*recession*) as the negative of the three-month moving average of these two monthly series, so a high value means the economic growth rate is lower.

The regression specification for our analysis, then, is the following

$$\begin{aligned} Y_{i,t} = \alpha + \beta\ commitment_{i,t-1} &+ \delta\ recession_{i,t} + \gamma\ commitment_{i,t-1} \cdot recession_{i,t} \\ &+ B\ controls_{i,t-1} + \zeta_i + \xi_t + \epsilon_{i,t} \end{aligned} \tag{16}$$

where the outcome variable is $Y_{i,t}$ is the change in debt from $t-1$ to t, scaled as described above.[31] The unit observation is a firm-quarter. Previous studies show that unobserved firm-level time-invariant factors explain a majority of variations in leverage ratios Lemmon, Roberts, and Zender (2008). Hence, we include firm fixed effects. We also include fiscal-year fixed effects. Standard errors are clustered at the firm level and are robust to

[30] Also following the literature, we exclude financial firms and regulated utilities. We also exclude firm quarters with $Q > 10$ or $Q < 0$.

[31] Under our model, firms adjust their debt continuously as their own output varies. Since output will be correlated with the state of economy, we also run versions of the regression that control for contemporaneous (or lagged) changes in firm sales. Inferences are unaffected and the point estimate of γ are similar. These specifications are omitted for brevity.

heteroscedasticity.

The hypothesis of interest is that $\gamma < 0$: the difference in debt issuance between firms with and without commitments is more negative when the aggregate growth rate of the economy is lower. Table 8 presents the results. The top panel uses the composite CFNAI recession measure; the bottom panel uses the Production and Income sub-index.

In both panels, with either covenant measure, and using each of the three scalings for debt changes, the coefficient on the interaction term is negative and statistically significant at the 5% or 1% level. Consistent with prior literature, covenant restrictions are unconditionally associated with greater reductions in debt. Consistent with Halling, Yu, and Zechner (2016), debt issuance is also overall countercyclical, i.e., more positive in recessions. However, firms with greater debt protection follow the opposite pattern, reducing debt in bad times.

Caption to Table 8. The table reports results of panel regressions of quarterly debt changes on measures of debt protection and economic conditions. The first three columns use the Murfin (2012) measure of covenant strictness. The next three columns use the log number of covenants. The covenant measures are lagged by one quarter. For each measure, three standardizations of debt changes are used as the dependent variable, following Welch (2011). In Panel A, the economic conditions variable (Recession) is the negative of the three-month moving average of the Chicago Fed National Activity Index (CFNAI). In Panel B, the variable is constructed likewise from the CFNAI Production and Income sub-series. Control variables are as defined in the text. Robust standard errors are clustered at firm level and are shown in parentheses. Asterisks (*,**,***) denote significance at the 10%, 5%, and 1% level, respectively.

Regarding the economic significance of the interaction effect, consider the change in implied debt issuance when the economy goes from one standard deviation above trend to one standard deviation below trend. Using the numbers from Table 7, this corresponds to an increase of 1.528 in the main recession measure. From the point estimates in column (1) of Panel A, a firm with well protected debt (strictness one standard deviation above the mean) would be expected to decrease debt by .00086 times assets, whereas a firm with unprotected debt (strictness one standard deviation below the mean) would increase debt by .00149 times assets. The difference in responses is 0.235% of assets per quarter or almost 1% per year. Viewed as an aggregate effect, this magnitude is economically meaningful. During the recent financial crisis, for example, in roughly two years from trough to peak back to trough, the economy's net debt to total book assets moved up and then down by approximately 4 percentage points,[32] while the CFNAI index dropped and then recovered by approximately four standard deviations. Thus, the effects estimated in the regression suggest that the contracting friction studied here could potentially play a significant role in aggregate financial cycles.

As a robustness test, Table 9 repeats the regressions replacing the measure of economic activity with measures of aggregate firm valuations. These tests directly confront the model's implication that expropriation incentives scale inversely with valuation multiples. Hence here the "recession" or "bad times" measure is the reciprocal of economy-wide estimates of Tobin's Q. The top panel computes the value-weighted average of firm-level Q for each COMPUSTAT firm each quarter. The results again strongly support the hypothesis of a negative interaction coefficient. The bottom panel uses the "bond-market Q" measure of Philippon (2009), which uses aggregate bond valuation data to impute Q from a structural model. This series only goes through 2007, reducing the power of the test, but also affirming that the findings here are not driven by the subsequent financial crisis. Using the Murfin proxy for commitment, the sample period only encompasses 1990-2007 and there is very little variation in the Q measure during this time. Still, all the point estimates are all negative, and two cases attain statistical significance at the 10% level. With the log covenant proxy for commitment, the number of observations almost doubles. Now, although the point estimates are actually reduced, they are highly statistically significant.

[32]The calculation uses the Federal Reserve Z.1 data for the U.S. nonfinancial corporate sector.

Caption to Table 9. The table reports results of panel regressions of quarterly debt changes on measures of debt protection and economic conditions. The first three columns use the Murfin (2012) measure of covenant strictness. The next three columns use the log number of covenants. The covenant measures are lagged by one quarter. For each measure, three standardizations of debt changes are used as the dependent variable, following Welch (2011). In Panel A, the economic conditions variable (Recession) is the inverse of the value-weighted average of firm-level Tobin's Q estimate. In Panel B, the variable is the inverse of the "bond market Q" measure of Philippon (2009). Control variables are as defined in the text. Robust standard errors are clustered at firm level and are shown in parentheses. Asterisks (*,**,***) denote significance at the 10%, 5%, and 1% level, respectively.

Summarizing, the results here confirm a novel prediction of our model, and constitute an addition to the empirical literature on covenant use. We document that firms with less protected debt tend to issue more debt in bad times. This is consistent with the interpretation that incentives to exploit existing creditors are higher in those times, resulting in a countercyclical component to the leverage of firms without commitment.

6. Conclusion

This paper contributes to the post-crisis literature that seeks to analyze the real effects of (corporate, household, and public) financial policies, and to highlight the frictions that influence those policies. Recent research has identified an underappreciated friction: the potential inability to commit managers to firm-maximizing, as opposed to equity-maximizing, debt policies. We extend this line of research and offer new insights into the dynamics of the problem.

Our setting allows us to quantify the private and social costs of lack of capital structure commitment under generalized preferences and with time-varying economic and firm parameters. We show that, both in the cross-section and time-series, expropriation incentives rise when firm valuations are low. Hence, without commitment, leverage can be countercyclical. This dynamic amplifies the distortionary effect of excess debt on aggregate risk. We present empirical evidence supportive of the prediction that firms with unprotected debt increase their borrowing in bad times.

An important next step in our understanding of the commitment problem is to describe (and empirically characterize) the contracting technology that dictates the supply side of monitoring and enforcement services. In the real world, some firms find it cost-effective to commit managers to financial policies via stringent covenants, while others do not – despite the direct savings in interest costs and the (larger) gain in private surplus. Our model offers a full account of those gains, as well as the potential welfare gains. Before drawing policy inferences from the analysis, however, we need to also understand the real resources required to implement commitment.

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Table 7: Summary Statistics

VARIABLES	N	Mean	Median	Std dev	N	Mean	Median	Std dev
	Sample 1: June 1981 - April 2017				Sample 2: February 1990 - March 2017			
Δ financial debt/total assets	96,920	0.007	0.000	0.063	57,889	0.007	0.000	0.060
Δ financial debt/fin debt+B equity	96,920	0.009	0.000	0.096	57,889	0.010	0.000	0.091
Δ financial debt/fin debt+M equity	96,920	0.004	0.000	0.058	57,889	0.004	0.000	0.054
Log(Total Assets)	96,920	6.409	6.448	1.886	57,889	6.770	6.829	1.857
Q	96,920	1.710	1.414	0.976	57,889	1.752	1.467	0.962
Cash Flow	96,920	0.016	0.021	0.040	57,889	0.019	0.023	0.037
Asset Tangibility	96,920	0.302	0.227	0.240	57,889	0.303	0.228	0.237
Commitment	96,920	0.866	1.099	0.473	57,889	0.196	0.143	0.192
-Chicago Fed National Activity Index	431	0.089	-0.037	0.764	326	0.152	0.043	0.726
-Production and Income Index	431	0.013	-0.027	0.285	326	0.022	-0.027	0.278

The table reports summary statistics for firm-quarters for which each commitment proxy can be computed. In Sample 1, Commitment is the maximum of the log number of covenants across all active syndicated loan packages. In Sample 2, Commitment is the maximum of the Murfin (2012) measure of covenant strictness across all active syndicated loan packages.

Table 8: Debt Issuance and Commitment

Panel A: Composite CFNAI Recession Measure

	(1)	(2)	(3)	(4)	(5)	(6)
	$\frac{\Delta\text{financial debt}}{\text{total assets}}$	$\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	$\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$	$\frac{\Delta\text{financial debt}}{\text{total assets}}$	$\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	$\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$
Covenant Strictness	-0.022***	-0.033***	-0.017***			
	(0.003)	(0.004)	(0.002)			
Strictness × Recession	-0.004**	-0.006**	-0.004**			
	(0.002)	(0.003)	(0.002)			
Log(#Covenants)				-0.002**	-0.003**	-0.002***
				(0.001)	(0.001)	(0.001)
Log(#Covenants)×Recession				-0.002***	-0.003***	-0.003***
				(0.001)	(0.001)	(0.001)
Recession	0.001	0.002*	0.001*	0.002***	0.002**	0.002***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Log(Total Assets)	-0.012***	-0.017***	-0.010***	-0.010***	-0.015***	-0.009***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Q	0.007***	0.010***	0.003***	0.006***	0.009***	0.003***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
Cash Flow	0.028*	0.028	0.031**	0.023**	0.035**	0.025**
	(0.014)	(0.023)	(0.013)	(0.010)	(0.017)	(0.010)
Asset Tangibility	0.035***	0.053***	0.033***	0.034***	0.049***	0.033***
	(0.007)	(0.010)	(0.006)	(0.005)	(0.008)	(0.005)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	55,009	55,009	55,009	91,173	91,173	91,173
R-squared	0.083	0.077	0.075	0.080	0.075	0.071

Panel B: Production & Income Recession Measure

	(1)	(2)	(3)	(4)	(5)	(6)
	$\frac{\Delta\text{financial debt}}{\text{total assets}}$	$\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	$\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$	$\frac{\Delta\text{financial debt}}{\text{total assets}}$	$\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	$\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$
Covenant Strictness	-0.023***	-0.034***	-0.018***			
	(0.003)	(0.004)	(0.002)			
Strictness × Recession	-0.010**	-0.016**	-0.010**			
	(0.004)	(0.007)	(0.004)			
Log(#Covenants)				-0.002***	-0.003**	-0.003***
				(0.001)	(0.001)	(0.001)
Log(#Covenants)×Recession				-0.004**	-0.006**	-0.005***
				(0.002)	(0.002)	(0.002)
Recession	0.003**	0.006**	0.003***	0.004**	0.006**	0.005***
	(0.001)	(0.002)	(0.001)	(0.002)	(0.003)	(0.002)
Log(Total Assets)	-0.012***	-0.017***	-0.010***	-0.010***	-0.015***	-0.009***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Q	0.007***	0.010***	0.003***	0.006***	0.009***	0.003***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
Cash Flow	0.028**	0.028	0.031**	0.023**	0.036**	0.026***
	(0.014)	(0.023)	(0.013)	(0.010)	(0.017)	(0.010)
Asset Tangibility	0.035***	0.053***	0.033***	0.034***	0.049***	0.033***
	(0.007)	(0.010)	(0.006)	(0.005)	(0.008)	(0.005)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	55,009	55,009	55,009	91,173	91,173	91,173
R-squared	0.083	0.077	0.075	0.080	0.075	0.071

Table 9: Debt Issuance and Commitment

Panel A: Aggregate Q

	(1) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(2) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(3) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$	(4) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(5) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(6) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$
Covenant Strictness	-0.001	-0.004	0.004			
	(0.011)	(0.017)	(0.010)			
Strictness × Recession	-0.045**	-0.064*	-0.047**			
	(0.022)	(0.033)	(0.020)			
Log(#Covenants)				0.008**	0.009	0.007**
				(0.004)	(0.006)	(0.003)
Log(#Covenants) × Recession				-0.022***	-0.027**	-0.020***
				(0.007)	(0.011)	(0.007)
Recession	0.014	0.029*	0.007	0.016*	0.025*	0.011
	(0.010)	(0.015)	(0.010)	(0.010)	(0.015)	(0.009)
Log(Total Assets)	-0.012***	-0.017***	-0.010***	-0.010***	-0.015***	-0.009***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Q	0.007***	0.010***	0.003***	0.006***	0.009***	0.003***
	(0.001)	(0.001)	(0.000)	(0.001)	(0.001)	(0.000)
Cash Flow	0.028*	0.028	0.031**	0.023**	0.035**	0.025***
	(0.014)	(0.023)	(0.013)	(0.010)	(0.017)	(0.010)
Asset Tangibility	0.036***	0.053***	0.033***	0.034***	0.049***	0.033***
	(0.007)	(0.010)	(0.006)	(0.005)	(0.008)	(0.005)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	55,009	55,009	55,009	91,173	91,173	91,173
R-squared	0.083	0.077	0.075	0.080	0.075	0.071

Panel B: Bond Market Q

	(1) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(2) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(3) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$	(4) $\frac{\Delta\text{financial debt}}{\text{total assets}}$	(5) $\frac{\Delta\text{financial debt}}{\text{fin. debt+B equity}}$	(6) $\frac{\Delta\text{financial debt}}{\text{fin. debt+M equity}}$
Covenant Strictness	0.031	0.035	0.041			
	(0.035)	(0.052)	(0.033)			
Strictness × Recession	-0.088*	-0.115	-0.092*			
	(0.052)	(0.077)	(0.049)			
Log(#Covenants)				0.038***	0.046***	0.036***
				(0.011)	(0.016)	(0.010)
Log(#Covenants) × Recession				-0.061***	-0.075***	-0.059***
				(0.016)	(0.024)	(0.015)
Recession	-0.012	-0.023	-0.025	0.045**	0.049	0.041*
	(0.023)	(0.034)	(0.022)	(0.022)	(0.035)	(0.022)
Log(Total Assets)	-0.019***	-0.028***	-0.016***	-0.016***	-0.023***	-0.013***
	(0.002)	(0.003)	(0.002)	(0.001)	(0.002)	(0.001)
Q	0.007***	0.009***	0.003***	0.006***	0.008***	0.003***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.000)
Cash Flow	0.046**	0.058**	0.042**	0.027**	0.051**	0.030**
	(0.018)	(0.030)	(0.017)	(0.013)	(0.022)	(0.013)
Asset Tangibility	0.057***	0.082***	0.048***	0.049***	0.066***	0.045***
	(0.011)	(0.016)	(0.010)	(0.008)	(0.012)	(0.007)
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes
Observations	30,364	30,364	30,364	55,246	55,246	55,246
R-squared	0.114	0.106	0.101	0.105	0.097	0.092

Appendix

A. Proofs

This appendix provides the proofs of the results in Section 2. Since Proposition 1 is a special case of Proposition 2, it suffices to prove the latter. We first establish the two lemmas on default policy.

Proof of Lemma 1.

The lemma asserts that the optimal default policy for equity holders is to abandon following a jump to Y_t if and only if the value of the firm is below the pre-jump value of debt, F_{t-}. To see this, if equity holders do not abandon, then their optimal debt policy at t is to adjust to the new quantity C_t whose value is F_t. If they do so, they repay the difference $F_{t-} - F_t > 0$ to debt holders, and their claim is now worth $V_t - F_t$. They will do this if and only if the debt repayment is less than the value they receive:

$$V_t - F_t > F_{t-} - F_t \iff V_t > F_{t-}$$

as asserted.

From this observation, it follows that we can link the optimal leverage ratio with the critical default threshold. Default occurs iff $V_t \leq F_{t-}$. So, if equality holds, we have

$$e^{\underline{\varphi}} \equiv \frac{Y_t}{Y_{t-}} = \frac{V_t}{V_{t-}} = \frac{F_{t-}}{V_{t-}} = \frac{pc}{v}.$$

Here the middle equality uses the assumption that, prior to default, firm value is linear in output. The final equality uses the assumptions that, prior to default, C is linear in Y and F is linear in C.

QED

Proof of Lemma 2. As above, equity holders will not abandon the firm upon a switch of states (from $s = 1$ to $s = 2$, say) if and only if the amount they owe to creditors is less than the net equity they will have upon payment, or $V(2) > F(1)$. Using the previous lemma, we can write $F(1) = e^{\underline{\varphi(1)}} V(1)$. The conclusion that default occurs if and only if $v(2)/v(1) < e^{\underline{\varphi(1)}}$ then follows given the linearity $V(s) = v(s)Y$ because Y is assumed independent of the state switching process. (The result for switches from 2 to 1 follows by symmetry.)

QED

Turning to Proposition 2, we assume the form of the pricing kernel in (12) and we derive the equations satisfied by the bond and firm value. The proof conjectures and verifies that linear forms solve these equations.

Proof of Proposition 2.

First consider the debt claim whose value is F and whose coupon amount is C. The price of a claim to $1/C$ units of the debt is denoted p. The proposition assumes that we are given a default policy (pair) $\underline{\varphi}(s)$ determining the exit jump threshold, and that these are constants (not functions of Y). Under this assumption, it is reasonable to conjecture that, absent default, p is not a function of Y since jumps that do not trigger default leave the firm exactly as far from the threshold as before (and the jump size distribution is independent of Y).

Let T denote the sooner of the firm's default time or the repayment time of the claim. (The firm may retire debt at any time by repurchasing at the open market price, and we may assume that it does so *pro rata* across outstanding units.) Then on $[0, T)$, the price p obeys the canonical equation that requires that its instantaneous payout per unit time (in this case, 1) times the pricing kernel Λ equals minus the expected change of the product process $p\Lambda$. Using Itô's lemma for jumping processes to expand the expected change implies

$$\left(\eta + \tfrac{1}{2}\lambda[(u^{-\gamma}\mathrm{E}_{\mathrm{t}}\left[\frac{p^{+}}{p}\right] - 1) + (d^{-\gamma}\mathrm{E}_{\mathrm{t}}\left[\frac{p^{-}}{p}\right] - 1)]\right) p(s) + \omega(s)[\xi(s)p(s') - p(s)] = -1. \tag{17}$$

Here s and s' represent current and alternative regimes, respectively, and $\frac{p^{+}}{p}$ and $\frac{p^{-}}{p}$ denote the fractional changes in p conditional on an up and down jump, respectively. (The notation supresses the posible dependence of η, λ, α, u, and d on s.) Next, rewrite the left side using the fact that the expected growth rate of the pricing kernel is minus the riskless rate:

$$r(s) = -\eta(s) - \tfrac{1}{2}\lambda(s)[(u(s)^{-\gamma} - 1) + (d(s)^{-\gamma} - 1)] - \omega(s)(\xi(s) - 1) \tag{18}$$

to get

$$[-r - \tfrac{1}{2}\lambda d^{-\gamma}(1 - \mathrm{E}_{\mathrm{t}}\left[\frac{p^{-}}{p}\right])]p(s) + \omega(s)\xi(s)[p(s') - p(s)] = -1. \tag{19}$$

where we have used the conjecture that outside of default p is not affected by jumps to put $\frac{p^+}{p} = 1$. To evaluate the down-jump expected change in p, we integrate over the jump-size distribution. If the jump in output, $\frac{Y^-}{Y}$ is greater than $\exp(\underline{\varphi})$, then $\frac{p^-}{p} = 1$. For worse jumps, our assumption is that creditors gain the rights to α times the cash flow stream. This is worth $V(\alpha Y^-)$ per unit claim, so $V(\alpha Y^-)/C(Y)$ to holders of p. Hence the recovery fraction is $\frac{p^-}{p} = V(\alpha Y^-)/(pC) = V(\alpha Y^-)/F(Y) = e^{-\underline{\varphi}}\ V(\alpha Y^-)/V(Y)$, where the last equality uses lemma 1. Using the linearity of V (to be verified below), the recovery fraction is just $\alpha e^{x-\underline{\varphi}}$ where x is the jump size $\log(Y^-/Y)$. All together then, $\mathrm{E_t}\left[\frac{p^-}{p}\right]$ is,

$$\int_{\underline{\varphi}}^{0} g^-(|x|)\ dx + \alpha e^{-\underline{\varphi}} \int_{-\infty}^{\underline{\varphi}} e^x\ g^-(|x|)\ dx,$$

or, using the notation defined in the text,

$$= G^-(|\underline{\varphi}|) + \alpha\ H(\underline{\varphi})e^{-\underline{\varphi}}.$$

Hence, (19) becomes

$$\left[r + \tfrac{1}{2}\lambda d^{-\gamma}(1 - [G^-(|\underline{\varphi}|) + \alpha\ H(\underline{\varphi})e^{-\underline{\varphi}}])\right] p(s) - \omega(s)\xi(s)[p(s') - p(s)] = 1. \tag{20}$$

This is equivalent to the system (13).

If the system has a strictly positive solution, this verifies that p is not a function of C and therefore $F = pC$. Similarly the solution is not a function of Y, verifying that conjecture.

Next, consider the valuation of the whole firm. Again, we are taking as given the default policy $\underline{\varphi}$. By Lemma 1, this gives us $e^{\underline{\varphi}} = F/V$ prior to default. And by the result just shown that $F = pC$, it follow that taking the default threshold as given is equivalent to taking the debt amount as a known function of V: $C = Ve^{\underline{\varphi}}/p$.

The after-tax cashflow stream to the firm prior to default is $(1-\tau)Y + \tau C$. Proceeding as above, we equate this quantity times Λ to minus the drift of $V\Lambda$, which yields

$$-\eta V - \mu Y\frac{\partial V}{\partial Y} - \tfrac{1}{2}\lambda[(u^{-\gamma}\mathrm{E_t}\left[V^+\right] - V) + (d^{-\gamma}\mathrm{E_t}\left[V^-\right] - V)] - \omega[\xi V' - V] = (1-\tau)Y + \tau C. \tag{21}$$

(This expression supresses the dependence on the current state s and denotes the value of V in the other state as V'.) We now look for a linear solution $V = v(s)Y$. In that case, cancelling a factor Y, the right side can be written

$$v\ \left(-\eta-\mu-\tfrac{1}{2}\lambda[(u^{-\gamma}U-1)+(d^{-\gamma}D-1)]\right)-\omega[\xi v'-v]$$

where we have defined $U(s)$ and $D(s)$ as $\mathrm{E}[Y^+/Y]$ and $\mathrm{E}[Y^-/Y]$, respectively. Using $C=Ve^{\underline{\varphi}}/p$, the left side is now

$$(1-\tau)+\tau v\ e^{\underline{\varphi}}/p.$$

Next, plugging in the expression for r in (18) and $\tilde{\mu}$ in (3), the right side becomes

$$v\ \left(+r-\tilde{\mu}-\tfrac{1}{2}\lambda[u^{-\gamma}(U-u)+d^{-\gamma}(D-d)]\right)-\omega\xi[v'-v].$$

Bringing the $e^{\underline{\varphi}}$ term to the right side, the above expression is equivalent to the system (14) in the text. If the coefficent matrix is postive definite, then there is a unique positive solution to the system, verifying the linearity conjecture.

Last, we note the expressions for U and D are:

$$U=\int_{\underline{\varphi}}^{0}\ e^{x}\ g^{+}(|x|)\ dx,$$

$$D=\int_{\underline{\varphi}}^{0}\ e^{x}\ g^{-}(|x|)\ dx+\alpha e^{-\underline{\varphi}}\ \int_{-\infty}^{\underline{\varphi}}\ e^{x}\ g^{-}(|x|)\ dx$$

which coincide with the definitions in (8) and (7).

QED

Turning to general equilibrium, the notation Y now denotes aggregate output, whose dynamics are derived in the text.

Proof of Proposition 3.

Given the aggregator function $f(C,J)$, the Bellman equation for J tells us that $\mathrm{E}[dJ]+f(C,J)\ dt=0$. Under the conjectured form for $J=J(s,Y)$, we have $\mathrm{E}[dJ]/J=$

$$(1-\gamma)\left[\mu(s)+\tfrac{1}{2}\lambda[(u(s)^{1-\gamma}-1)+(d(s)^{1-\gamma}-1)]\right]+\omega(s)(\frac{j(s')}{j(s)}-1)$$

Dividing $f(C,J)$ by J and using $Y=C$, we get the two terms[33]

$$\beta\theta j(s)^{-\frac{1}{\theta}} - \beta\theta.$$

Adding these to the $\mathrm{E}[dJ]/J$ terms and multiplying by j gives:

$$\beta\theta j(s)^{1-\frac{1}{\theta}} - \beta\theta j(s) + (1-\gamma)\left[\mu j(s) + \tfrac{1}{2}\lambda[(u(s)^{1-\gamma}-1)+(d(s)^{1-\gamma}-1)]j(s)\right] + \omega(s)(j(s')-j(s)) = 0.$$

This is the algebraic system referred to in the proposition, whose solution gives the constants $j(1), j(2)$.

Given solutions for J and C, Duffie and Skiadas (1994) show that the pricing kernel under stochastic differential utility is

$$\Lambda_t = e^{\int_0^t f_J(C_u,J_u)\,du} f_C(C_t,J_t).$$

Here, using $C=Y$ and the solution for J, we get $f_C(C,J)=\beta\, j(s)^{1-\frac{1}{\theta}}\, Y^{-\gamma}$, and

$$f_J(C,J) = \beta\theta\left[(1-\frac{1}{\theta})\, j(s)^{-\frac{1}{\theta}} - 1\right].$$

The propostion then just evaluates the dynamics $d\Lambda/\Lambda$ from these expressions.

The integral term contributes an f_J term to the drift. To this we add df_C/f_C, which is

$$-\gamma\mu_Y\; dt + d\left\{\sum_{j=1}^{\mathcal{J}_t}\left((u^{-\gamma}-1)1_{\{j,+\}} + (d^{-\gamma}-1)1_{\{j,-\}}\right)\right\}$$
$$+d\left\{\sum_{i=1}^{\mathcal{I}_t}\left(\left(\left[\frac{j(s')}{j(s)}\right]^{1-\frac{1}{\theta}}-1\right)1_{\{i,s'\}} + \left(\left[\frac{j(s')}{j(s)}\right]^{1-\frac{1}{\theta}}-1\right)1_{\{i,s\}}\right)\right\}.$$

The expression for η in the proposition is f_J plus the drift contribution from the previous expression. The ratios in the last term, which represent the fractional changes of f_C on a change in state, are are the quantities denoted $\xi(s)$. The expression for riskless rate is minus the expected change of $d\Lambda/\Lambda$.

QED

Returning to the single-regime case, we next prove Corollary 2.1.

[33] Recall $f(C,J) \equiv \frac{\beta C^\rho/\rho}{((1-\gamma)\;J)^{1/\theta-1}} - \beta\theta J$.

Proof. The corollary considers the case $\alpha = 0$. So, using the expressions in Proposition 1, take the derivative of v with respect to $\underline{\varphi}$, and set it equal to the derivative of $\bar{c}\ p$. Bringing all multiplicative factors to the right side, and letting $y = 1/p$, left side is simply

$$(1-\tau)\tilde{\ell}_d\ g^-(|\underline{\varphi}|) - \tau y.$$

This represents the net marginal cost of debt: the first term corresponds to the marginal default cost and the second to minus the marginal tax benefit. Under committment, the firm looks for a zero of this function to find the optimal $\underline{\varphi}$. To ensure one exists, and is unique, we impose conditions sufficent for the function to be monotonically increasing, and negative at $\underline{\varphi} = -\infty$ and positive at $\underline{\varphi} = 0$. Here $y = r + \tilde{\ell}_d\ (1 - G^-)$. So differentiating again, monotonicity is equivalent to the first condition assumed in the statement of the corollary: $\frac{1}{g^-(x)}\frac{dg^-(x)}{dx} \leq -\frac{\tau}{1-\tau}$. For the limit at infinity, g^- goes to zero, and y goes to r so the expression's limit is $-\tau r$ which is negative if r is positive. At $\underline{\varphi} = 0$, G^- is zero, so positivity is equivalent the second assumption $g^-(0) > \frac{\tau}{1-\tau}\frac{r+\tilde{\ell}_d}{\tilde{\ell}_d}$.

(In the numerical work, we assume G^- is a gamma distribution with mean σ and variance $L^2\sigma^2$, with $L \geq 1$. For this parameterization, the assumed conditions are satisfied when $L\sigma < \frac{1-\tau}{\tau}$.)

Next we turn to the right side of the first order condition. After some rearrangement, this is

$$(1-\tau)\bar{c}\ \tilde{\ell}_d\ g^-(|\underline{\varphi}|)\ (p/v)^2\ e^{-\underline{\varphi}}$$

which is positive for any $\bar{c} > 0$. Thus, if there is a $\bar{c}$ that satisfies the condition of a no-commitment equilibrium, the left side must intersect the right side and do so at a positive value of both. Since the left side has been shown to be monotonically increasing, it follows that the intersection is to the right of the firm-value maximizing point, which is the zero of the left side. A higher (less negative) value of $\underline{\varphi}$ corresponds to higher market leverage because of the optimal default condition that equates $e^{\underline{\varphi}}$ to $F/V = cp/v$.

We also note that, if $\bar{c}$ is an equilibrium solution, then we may use the latter condition, $\bar{c} = e^{\underline{\varphi}}\ v/p$ to substitute it out in the right-side expression above to obtain simply

$$(1-\tau)\ \tilde{\ell}_d\ g^-(|\underline{\varphi}|)\ p/v.$$

This is the expression referred to in the text as the expropriation incentive.

$\square$

It remains to prove Corollary 2.2.

Proof. To prove the existence and uniqueness of the no-commitment equilibrium, we need to find a unique fixed point $\bar{c}^*$ such that $c(\bar{c}^*) = \bar{c}^*$. Consider the marginal incentive function $M(c, \bar{c}) = \frac{dv}{dc} - \bar{c}\frac{dp}{dc}$. If we evaluate the function at $c = \bar{c}$, it is the marginal incentives to alter debt when we start off with debt $c = \bar{c}$. If there is a unique $\bar{c}^*$ such that $m(\bar{c}^*) = M(c^*(\bar{c}^*), \bar{c}^*) = 0$, then the unique no-commitment equilibrium exists.

If $\bar{c}$ is large enough, the objective function $v - \bar{c}p$ is always negative. Denote $\bar{c}_h$ as the highest $\bar{c}$ with nonnegative equity value. The domain of the fixed point $\bar{c}^*$ is $(0, \bar{c}_h)$.

Here are four conditions we must establish to get the unique NC equilibrium.

(i) $m(c) > 0$ when $c \to 0$,

(ii) $m(c')$ is negative for some high value $c' < \bar{c}_h$,

(iii) $m(c)$ is monotonically decreasing in c,

(iv) Suppose $\bar{c}^*$ is the fixed point. Then the Second Order Condition $\frac{dM(c(\bar{c}^*),\bar{c}^*)}{dc(\bar{c}^*)} < 0$ holds .

In the following discussion, we list all the assumptions needed to establish each condition.

Condition (i)

Given the assumptions in Corollary 2.1, we have condition (i). Intuitively, when the firm starts off with $\bar{c} = c = 0$, the marginal incentive being positive means it has the incentive to increase debt.

Condition (ii)

Assumptions needed to establish condition (ii) are the following:

1. $\frac{dc}{d\underline{\varphi}} > 0$ when $c \in (0, \bar{c}_h)$.

2. There exists a high value $\bar{c}' < \bar{c}_h$, with its corresponding $\underline{\varphi}, p, v$ to be $\underline{\varphi}', p', v'$, such that

$$(\frac{v'}{p'} - 1)(-p'^2\tilde{l}_d g^-(|\underline{\varphi}'|)) + \frac{\tau}{1-\tau}v' < 0$$

Since $\frac{d\bar{c}}{d\underline{\varphi}} > 0$, the marginal incentive function $m(\bar{c}') = \frac{dv'}{d\bar{c}'} - \bar{c}'\frac{dp'}{d\bar{c}'}$ can be rewritten as $m(\underline{\varphi}') = M(\underline{\varphi}', e^{\underline{\varphi}'}\frac{v'}{p'}) = \frac{dv'}{d\underline{\varphi}'} - e^{\underline{\varphi}'}\frac{v'}{p'}\frac{dp'}{d\underline{\varphi}'} = e^{\underline{\varphi}'}\frac{v'}{p'}\Big[(\frac{v'}{p'} - 1)\frac{dp'}{d\underline{\varphi}'} + \frac{\tau}{1-\tau}v'\Big] = e^{\underline{\varphi}'}\frac{v'}{p'}\Big[(\frac{v'}{p'} -$

$1)(-p'^2\tilde{l}_d g^-(|\underline{\varphi}'|)) + \frac{\tau}{1-\tau}v'\Big] < 0$. The economic intuition of condition (ii) is that for some high value $\overline{c}' < \overline{c}_h$, the firm has the incentive to reduce its debt. From (i) (ii) and continuity of $m(c)$ we conclude there exists a $\overline{c}^*$ where $m(\overline{c}^*) = 0$.

Condition (iii)

Assumptions needed to establish condition (iii) are the following:

1. $\frac{dc}{d\underline{\varphi}} > 0$ when $c \in (0, \overline{c}_h)$.

2. $\tilde{l}_d g^-(|\underline{\varphi}|)[1 - \frac{v}{p}e^{\underline{\varphi}}] + \frac{\tau}{1-\tau}e^{\underline{\varphi}}\frac{v}{p^2} > 0$.

3. $v > p$.

4. $-\frac{dg^-(|\underline{\varphi}|)}{d\underline{\varphi}} < -2p\tilde{l}_d\ g^-(|\underline{\varphi}|)^2$.

5. $(\frac{\tau}{1-\tau} - p\tilde{l}_d g^-(|\underline{\varphi}|)) < 0$.

With $\frac{dc}{d\underline{\varphi}} > 0, c \in (0, \overline{c}_h)$ we can redefine the marginal incentive function $m(\underline{\varphi}) = M(\underline{\varphi}, e^{\underline{\varphi}}\frac{v}{p}) = \frac{dv}{d\underline{\varphi}} - e^{\underline{\varphi}}\frac{v}{p}\frac{dp}{d\underline{\varphi}} = e^{\underline{\varphi}}\frac{v}{p}\Big[(\frac{v}{p} - 1)\frac{dp}{d\underline{\varphi}} + \frac{\tau}{1-\tau}v\Big]$. Suppose $v > 0, p > 0$, the sign of $\frac{dm}{d\underline{\varphi}}$ is the same as

$$\frac{d\Big[(\frac{v}{p} - 1)\frac{dp}{d\underline{\varphi}} + \frac{\tau}{1-\tau}v\Big]}{d\underline{\varphi}}$$
$$= \underbrace{\Big[-\frac{v}{p^2}\frac{dp}{d\underline{\varphi}} + \frac{1}{p}\frac{dv}{d\underline{\varphi}}\Big]}_{>0} \underbrace{\frac{dp}{d\underline{\varphi}}}_{<0} + \underbrace{(\frac{v}{p} - 1)}_{>0} \underbrace{\frac{d^2p}{d\underline{\varphi}^2}}_{<0} + \frac{\tau}{1-\tau} \underbrace{\frac{dv}{d\underline{\varphi}}}_{<0} < 0,$$

where

$$\Big[-\frac{v}{p^2}\frac{dp}{d\underline{\varphi}} + \frac{1}{p}\frac{dv}{d\underline{\varphi}}\Big] = \tilde{l}_d g^-(|\underline{\varphi}|)v[1 - \frac{v}{p}e^{\underline{\varphi}}] + \frac{\tau}{1-\tau}e^{\underline{\varphi}}\frac{v^2}{p^2},$$
$$\frac{d^2p}{d\underline{\varphi}^2} = -p^2\tilde{l}_d\frac{dg^-(|\underline{\varphi}|)}{d\underline{\varphi}} + 2p^3\tilde{l}_d^2 g^-(|\underline{\varphi}|)^2,$$
$$\frac{dv}{d\underline{\varphi}} = e^{\underline{\varphi}}\frac{v^2}{p}(\frac{\tau}{1-\tau} - p\tilde{l}_d g^-(|\underline{\varphi}|)).$$

Monotonicity of $m(c)$ ensures the uniqueness of the fixed point $\overline{c}^*$.

Condition (iv)

Condition (iv) is the sufficient condition for $\bar{c}^*$ to be the optimal solution to maximize the equity value $v - \bar{c}p$. It can be derived from condition (iii). From condition (iii) we know, for any $c \in (0, c')$, we have

$$\frac{dm}{d\underline{\varphi}} = \frac{d^2v}{d\underline{\varphi}^2} - c(\underline{\varphi})\frac{d^2p}{d\underline{\varphi}^2} \underbrace{-\frac{dc}{d\underline{\varphi}}\frac{dp}{d\underline{\varphi}}}_{>0} < 0.$$

Since $\bar{c}^*$ is also in $(0, c')$, and $-\frac{dc}{d\underline{\varphi}}\frac{dp}{d\underline{\varphi}}$ is positive, we can conclude that

$$\frac{dM(c(\bar{c}^*), \bar{c}^*)}{dc(\bar{c}^*)} = \frac{d^2v}{d\underline{\varphi}^{*2}} - c(\underline{\varphi}^*)\frac{d^2p}{d\underline{\varphi}^{*2}} < \frac{dm}{d\underline{\varphi}^*} < 0.$$

$\square$