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A Unified Approach for Cycle Service Levels

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This paper concerns the analysis and the enforcement of *cycle* service levels. We consider multi-stage supply chains that must respond to a stochastic customer's demand. They seek to construct integrated replenishment plans that satisfy strict stockout-oriented performance measures which apply across a multi-period planning horizon, hereby referred to as cycle. We develop a modeling and solution approach that can be uniformly applied to the ready rate, fill rate, and conditional expected stockout cycle service levels. We propose new planning models, formulate the constraints corresponding to the above cycle service levels, and, using the concepts of *service level sufficient* and *efficient* demand trajectories, we derive a deterministic reformulation of the original stochastic planning optimization problem. An extended computational study evaluates the efficiency of the solution approach on a real-life problem faced by a chemical supply chain, and demonstrates its applicability to standard and more complex supply chain networks. The scope of the approach is further widened by the fact that it can handle most types of dependency structures between random variables. We also analyze the degree of conservativeness of the three modeling approaches proposed for the ready rate service level, compare stagewise versus cycle service levels, and study the joint enforcement of cycle service levels limiting both the stockout probability and magnitude.

1. Introduction

1.1 Cycle Service Level

A stockout does not only cause the immediate profit loss of the canceled order, but it also harms the long term profitability since it decreases the likelihood of receiving new orders from the customers whose demand could not be satisfied (see [2] for a study about the effect of stockouts). This, combined with the increased globalization and competition, has exacerbated the need to enforce highly demanding service levels for supply chains, in particular for those operating in highly competitive, military environments [24, 32], or for those in which a supply shortage triggers huge costs to set up the production process and machines all over again. Such a scenario is illustrated by one of the 2007 Edelman Award Finalists, the calcium carbonate producer Omya Hustadmarmor [11], in which the negative impact of a stockout is described. Shortage management is a key supply chain performance driver and it triggers the enforcement of stockout-related cycle service levels. The importance of developing methods that allow for a better evaluation and control of the service level performance over a mid-term planning horizon is also discussed in [45].

In this paper, we analyze *cycle* service levels that limit the probability or the magnitude of a stockout across a multi-period planning horizon and study their enforcement by multi-stage supply chains. The notion of a cycle here must be understood as the duration of the entire planning horizon composed of a number of *interdependent* time-periods. It is very important to stress this point, since the term cycle is sometimes used to describe the duration of the replenishment process. Accordingly, Chopra and Meindl [9] (see also [5]) define the cycle service level as the "probability of not having a stockout in a replenishment cycle". As a marked difference to this, we define a cycle service level (see also [26, 28, 42]) as one that requires the probability or magnitude of a shortage occurring across the entire planning horizon to be lower than a small prescribed value. Hence, in the remainder of this paper, the concept of cycle service level (CSL) must be understood along the above definition.

A cycle service level also differs from a stagewise (i.e., one-period t) service level $p[t]$ that enforces non-stockout requirements at each period considered *independently* of each other. A cycle service level provides a representative measure of the responsiveness of the supply chain across the planning horizon [24], and ensures that, on average, $100p[t]\%$ of customers are satisfied all the time. A stagewise service level provides an expected value measure that reflects the steady-state nature of the supply chain. A high probability of not having a shortage at any of a number of periods does not strictly constrain the probability of not having a shortage over the planning horizon spanning across these periods [37]. It follows that the enforcement of a stagewise service level could result in a very low cycle service level, especially if the number of periods or entities in the supply chain is large [28]. To illustrate this, we consider a one-year planning horizon decomposed into 4-month periods. A stagewise (4-month) service level policy enforcing that

- the probability of not being in shortage in period 1 is at least equal to 90%,
- the probability of not being in shortage in period 2 is at least equal to 90%, and
- the probability of not being in shortage in period 3 is at least equal to 90%

guarantees a cycle service level (annual probability of no shortage) of only 72.9% (i.e., 0.9^3) in case of independence between quarters. In case of dependence between quarters, the cycle service level that can be guaranteed through the enforcement of stagewise service level constraints becomes much more difficult to evaluate, and is possibly lower than under the independence assumption.

Service levels are modeled with *chance* also called *probabilistic constraints* [35]. Constraints representing stagewise service levels take the form of *individual* probabilistic constraints, for which a deterministic equivalent can be easily derived using the quantile of the associated (univariate) probability distribution [35]. By contrast, constraints enforcing cycle service levels require the computation of multivariate probabilities, and they take the form of *joint* probabilistic constraints, for which it is much more challenging to derive good deterministic formulations [12, 26, 28, 35, 36].

Beyond the distinction between cycle and stagewise service levels, it is important to also differentiate service levels along the nature of shortages they aim at preventing. In this paper, we consider the *ready rate* service level which limits the *probability* of having a stockout, and the *fill rate* and the *conditional expected shortage* service levels, which both limit the shortage *quantities* (i.e., the expected fraction of stockout for the former and the conditional expected shortage for the latter). The ready rate service level is favored when customer satisfaction is primarily driven by the occurrence or not of a stockout, while its magnitude is secondary [23]. The fill rate service level is most often enforced when the focus is on limiting the expected proportion of demand that cannot be satisfied using the immediately available inventory [23]. The conditional expected stockout [1, 39] represents the quantity of products that will be short given that a stockout will occur. It is particularly important to compute and limit this value when it is necessary to order the amount of non-available product from a second source [1]. The reader is referred to [21, 23] for a discussion of the suitability of the various kinds of service levels.

1.2 Contributions and Literature Review

The major contribution, as well as novelty, of this study is that it provides decision makers with a modeling and solution approach enabling them to develop policies enforcing stockout-related CSL. A key feature of our approach is its wide applicability. It is applicable to to several types of CSL and to multiple formulations (i.e., three models are discussed for the ready rate CSL). Moreover, the proposed approach is not affected by any restrictive (independence) assumption about the random variables. Our approach can handle stationary as well as non-stationary random demand, which implies that it can account for demand seasonality.

First, we develop new models allowing for the construction of replenishment plans accounting for the requirements of three types of CSL that limit the probability or the magnitude of stockout across a multi-period planning horizon. The proposed models are multi-functional and define the optimal production, distribution, and inventory decisions. We also model the simultaneous enforcement of several of those service levels. The present study is an extension of [26, 28] which first introduced the concept of CSL. In [28], the authors focus on the ready rate CSL. They formulate the associated stochastic optimization problem for which they derive a deterministic reformulation using the concept of p-efficient demand trajectory. In [26], several preprocessing and column generation algorithms are proposed and evaluated to construct a replenishment plan enforcing the ready rate CSL. This study has a broader scope in that it develops and solves models representing new quantitative cycle service levels (fill rate and conditional expected stockout service levels in addition to the ready rate one). Moreover, it also proposes two approximate formulations for the ready rate CSL. The approximations are more conservative (i.e., enforce stronger requirements than those of the targeted ready rate CSL) than the model studied in [26, 28], but do not require the tackling of a joint (multivariate) probabilistic constraint. A comparison of the computational tractability and degree of conservativeness of the three ready rate CSL models is presented.

Second, we develop a *computationally tractable* and *general* solution method that can be applied uniformly to the stochastic optimization problems associated with the three CSLs mentioned above. The solution method rests on the concepts of *service level sufficient* and *efficient* demand trajectories which, respectively, represent a set of sufficient and minimal conditions that must be satisfied in order to attain the prescribed cycle service level. These types of demand trajectories are extensions of the p-efficient demand trajectory concept [28]. The service level efficient (resp., sufficient) demand trajectory is instrumental to derive a deterministic equivalent (resp., approximative) reformulation of the original stochastic planning optimization problem.

Third, the tractability of the proposed approach, the differences in reliability stemming from the enforcement of stagewise and cycle service levels, the risk-aversion character of the proposed models for the ready rate CSL, and the overall applicability of the approach, are evaluated through a computational study. At first, the approach is used on a real-life problem faced by a North American chemical supply chain to construct integrated replenishment plans that satisfy very strict service level requirements. Then, in order to appraise the extendibility of our approach to most multi-stage supply chains, we remove the assumptions particular to that supply chain and increase the dimensionality of the problem (i.e., larger supply chain network, transportation fleet, and number of realizations that the random demand can possibly take).

The review of the literature that follows focuses on models which explicitly account for the uncertainty in the design of production-distribution systems. Paschalidis et al. [33] derive a base-stock production policy for a multi-stage supply chain that faces a stochastic demand and whose one-period probability of stockout must be below a prescribed level. Hall and Potts [20] develop an integrated production-distribution model whose objective function accounts for customer service and transportation costs. The model assumes that products can be delivered on the spot, without any transportation time and does not consider routing decisions. Chen and Vairaktarakis [8] develop an integrated production-distribution model in which the goods are directly delivered from the producer to the end customer. A service level policy defining the time at which the products are supplied to the customers is derived. Cardós et al. [5] calculate the attained cycle service level in an (R, S) periodic review inventory system when the demand is discretely distributed. Yildirim et al. [46] use stochastic dynamic programming to build a production and sourcing plan over a multi-period horizon. Distribution decisions are not considered, and individual service level constraints that limit the probability of having a stockout at a single period are included. Tempelmeier [44] studies the uncapacitated single-item dynamic lot-sizing problem with stochastic period demands and backordering. Models that minimize the setup and holding costs, while including a service level constraint, are presented. Kutanoglu and Lohiyaa [25] develop an integrated base-stock inventory-transportation model for a single-echelon, multi-facility service parts logistics system with time-based service level constraints. The study shows the savings obtained through the integration of the inventory and transportation decisions. Nagar and Jain [31] use multi-stage stochastic programming to introduce a multi-period supply chain model for the launching of new products with uncertain demand. The model allows for the adjustment of the production plan as uncertainties are progressively resolved. The determination of the optimal safety stock levels needed to achieve predefined fill rate service levels in multi-stage networks confronted with uncertain demands is studied in [4]. A simulation approach is employed to solve the supply chain planning problem on a rolling horizon basis. We refer the reader to [7, 30, 40] for a more detailed review of production-distribution models and to [18, 41, 43] and the references therein for similar models considered in a stochastic context. Stochastic programming models for the attainment of various objectives pursued by supply chains are proposed in [13].

The present paper is organized as follows. In Section 2, we describe the probabilistic inventory-production-distribution model permitting the construction of replenishment plans that meet the requirements of CSL. In Section 3, we elaborate on the solution method. Section 4 comments the results of the computational study. Section 5 offers concluding remarks.

2. Probabilistic Replenishment Planning Model

We develop a replenishment planning model for the taking of integrated inventory, production and distribution decisions in a three-stage supply chain that faces a stochastic end-customer demand. The following properties of the stochastic planning model warrant further explanation.

First, the replenishment plan is integrated and multi-functional. The decision variables of the problem concern the: *(i)* production function (production levels of products at each supply chain node and period t); *(ii)* distribution function (scheduling and routing of the transportation fleet, quantity of products supplied to each node at each period); *(iii)* inventory function (inventory levels of products at each node and period).

Second, the proposed model does not attempt to quantify the cost of losing customers, but instead imposes, through the use of service level constraints, the minimization of the costs subject to the attainment of a prescribed service level, thus effectively limiting the risk of a loss of customers. This contrasts with a profit maximization approach where a penalty cost (the quantification of which is very difficult) for the unmet demand must be included in the objective.

Third, the model is developed for the enforcement of *cycle* service levels that must hold throughout the whole planning horizon which comprises a finite number of periods. Fourth, we do not accept the backlogging of the demand not satisfied from the on-hand stock. The rationale is that, very often, the lagged satisfaction of an order is not accepted by demanding customers, those having a greater negotiating power, or those suffering from high set up costs when there is a shortage [11]. Moreover, if the demand cannot be met with the on-hand inventory, the delay to satisfy it will sometimes be exceedingly long. Fifth, the constructed plan must be reproducible over the next planning horizon. Constraints preventing the so-called end-effect errors [16], i.e., "what is optimal over the short horizon may be suboptimal over the long run", are introduced in order to avoid the depletion of the inventories.

The following notations are used in the paper: T is the set of periods in the planning horizon and J is the set of distribution centers delivering to end-customers.

We denote by $d[j,t]$ the stochastic product demand that can take, at each time $t \in T$ and distributor $j \in J$, ℓ different levels $d^l[j,t]$ with probabilities: $\mathbf{p}^l[j,t] = P(d[j,t] = d^l[j,t]),\ l = 1,\dots,\ell$ such that $\sum_{l=1}^{\ell} \mathbf{p}^l[j,t] = 1$. Without loss of generality, we assume that the levels of the random variables are ordered: $d^{l-1}[j,t] < d^l[j,t], l = 2,\dots,\ell$. The random cumulative demand $\xi[j,t] = \sum_{t'=1}^{t} d[j,t']$ has a discrete probability distribution and can take up to ℓ^t (ℓ to power t) different levels at each period t.

The probabilistic planning model enforcing a cycle service level is formulated, in its most general form, as a stochastic mixed-integer programming (MIP) problem, and its compact formulation is

$$\min \quad f(x) \tag{1}$$

$$\text{subject to} \quad Ax \geq b \tag{2}$$

$$\omega \overset{(p)}{\succ} \xi \tag{3}$$

$$x = [x'\ x"] \in \mathcal{R}_+ \times \mathcal{Z}_+ \tag{4}$$

The vector of production, inventory, and distribution decision variables x can be partitioned into $[x'\ x'']$. Elements of x'' are restricted to be positive integer numbers. Elements of x' are real positive numbers, and encompass all the other decision variables. The symbol $\mathcal{R}_+$ refers to the appropriate multi-dimensional space of positive real vectors, while the symbol $\mathcal{Z}_+$ refers to the appropriate multi-dimensional space of positive integer vectors. The objective function (1) represents the total costs of the supply chain and is the summation of the inventory, production, and distribution costs. We denote by (2) the set of deterministic constraints, while the set of stochastic service level constraints is denoted by (3). The service level constraints are defined with respect to the cumulative demand ω (ω is a component of x) and supply ξ. The notation $\omega \overset{(p)}{\succ} \xi$ means that the cumulative supply must satisfy the cumulative demand in a way that allows the attainment of the cycle service level p. The supply chain does not intend to be able to handle all demand levels, since this would force its entities to keep very large safety stocks and to support excessive costs. Instead, the supply chain wants to respond to the random customer's demand with a given service level enforced through a service level constraint. We shall see in the next sections that the service level constraint takes different forms, depending on the type (ready rate, fill rate, conditional expected stockout) of CSL.

3. Unified Solution Method

We develop a solution method to construct integrated replenishment plans that satisfy the conditions imposed by stockout-related CSLs. The method can be uniformly applied to any types of CSLs (ready rate, fill rate, conditional expected stockout).

The solution method involves the derivation of a deterministic equivalent or approximation for the stochastic optimization problem (1)-(4). The deterministic reformulation is obtained using the concepts of service level sufficient and efficient demand trajectories.
The m^{th} demand trajectory D_{j_m} at distributor j is a $|T|$-dimensional vector defined [28] as

$$D_{j_m} = \left[\xi_m[j,1], \ldots, \xi_m[j,t], \ldots, \xi_m[j,|T|]\right]$$

whose components $\xi_m[j,t], t \in T$ are the cumulative demand quantities at distributor j until time t characterizing D_{j_m}. The notation $|T|$ refers to the cardinality of the set T.

Definition 1 *A demand trajectory D_{j_m} is said to be*

- *service level sufficient if the satisfaction of the demand quantities $\xi_m[j,t], t \in T$, characterizing D_{j_m}, ensures that $\omega \overset{(p)}{\succ} \xi$ holds, and*
- *service level efficient if the demand quantities $\xi_m[j,t], t \in T$ define the minimal requirements that must be satisfied for $\omega \overset{(p)}{\succ} \xi$ to hold.*

The minimality of the demand requirements is understood as follows. If the trajectory D_{j_m} is service level efficient, then there is no other trajectory D_{j_n} imposing less demanding requirements whose satisfaction would ensure that $\omega \overset{(p)}{\succ} \xi$ holds.
The corollaries below follow:

Corollary 1 *If a demand trajectory is service level efficient, than it is service level sufficient.*

The reverse is not true.

Corollary 2 *The set of constraints*

$$z[j,0] + \omega[j,t] \geq \xi_m[j,t], \ t \in T \ ,$$

where $\xi_m[j,t]$ is the cumulative demand quantity of the service level sufficient (resp., efficient) demand trajectory D_{j_m}, define sufficient (i.e., minimal) conditions for the attainment of the prescribed CSL.

The proof is straightforward from Definition 1 and the definition of the service level constraint.

The next question to settle relates to the cumulative demand quantities defining service level sufficient and efficient demand trajectories. For each type (ready rate, fill rate, conditional expected stockout) of CSL and modeling approach, we shall formulate deterministic constraints to represent the cumulative demand quantities characterizing service level sufficient and efficient demand trajectories. We shall show that, regardless of the type of CSL, there exists at least one service level efficient demand trajectory, and that the number of service level efficient and sufficient demand trajectories, as well as the values taken by the cumulative demand amounts characterizing them, is highly dependent on the type of CSL and the modeling approach employed.

3.1 Probability of Stockout

In this section, we consider the *ready rate*, also called *non-stockout*, cycle service level. This is an event-oriented performance criterion that requires the probability of having a shortage, at any point during the planning horizon, to be below a predefined level. The ready rate service level ensures that all customer orders arriving over the entire planning horizon will be completely satisfied from the available stock with a large probability.

The attainment of the ready rate CSL is modeled with joint probabilistic constraints [26, 28] ensuring that the probability of the joint fulfillment of a system of linear inequalities with dependent random right-hand side variables is above a prescribed probability level p, representing the ready rate CSL. We formulate one joint probability constraint for each distributor j:

$$P(z[j,0] + \omega[j,t] \geq \xi[j,t],\ t \in T) \geq p[j],\ j \in J \ . \tag{5}$$

These constraints require $\omega[j,t] + z[j,0]$ to be at least equal to $\xi[j,t]$, with some probability $p[j]$, for all possible realizations of the right hand side $\xi[j,t]$, and they give the supply chain the possibility to provide a differentiated service level $p[j]$ to each client or distributor j. Substituting (5) for (3) in (1)-(4), we obtain the stochastic integer programming problem

$$\begin{aligned} & \min c^T x \\ \text{subject to } & Ax \geq b \\ & P(z[j,0] + \omega[j,t] \geq \xi[j,t],\ t \in T) \geq p[j],\ j \in J \\ & x = [x'\ x''] \in \mathcal{R}_+ \times \mathcal{Z}_+ \end{aligned} \tag{6}$$

whose continuous relaxation is non-convex. The level of the random cumulative demand at time t is not independent of that at time $(t-1)$. Therefore, the joint probabilistic constraints have *dependent* random variables located in the right-hand side, and the probability $P(z[j,0]+\omega[j,t] \geq \xi[j,t],\ t \in T)$ is a $|T|$-variate one. We refer the reader to [28] for a detailed discussion of the reasons behind the formulation of the service level constraints in terms of the cumulative demand and supply as opposed to their formulation in terms of the demand and supply. In the next sub-sections, we derive three modeling approaches for the ready rate CSL.

3.1.1 p-Efficiency Model

The modeling approach described in this section allows the direct handling of the multivariate probability distribution and was proposed in [28]. It rests on the concept of the p-efficient point of a discrete probability distribution [35]. Let $p \in [0,1]$ be a probability level, and F be a discrete cumulative distribution function: $F(v) = P(v \geq \xi),\ \xi \in \mathcal{Z}_+^q$.

Definition 2 *A point $v \in \mathcal{R}^q$ is p-efficient [35] for the probability distribution function F if:*

1. $F(v) \geq p$, *and*
2. *There is no* $v' \leq v, v' \neq v$ *such that* $F(v') \geq p$.

Denoting by e_i the p-efficient point of the marginal distribution $F_i(\cdot)$, $i = 1, \ldots, q$, a direct consequence of Definition 2 is that

$$v \geq e = (e_1, \ldots, e_i, \ldots, e_q)$$

for every q-dimensional vector v such that $F(v) \geq p$.

Transposing the p-efficiency concept into the stochastic supply chain management problem discussed here, we define the ready rate (p) efficient demand trajectory as follows.

Definition 3 *A demand trajectory $D_{j_m} = \left[\xi_m[j,1], \ldots, \xi_m[j,|T|]\right]$ is ready rate p-efficient if*

1. *$F(D_{j_m}) \geq p$, and*

2. *There is no D_{j_n} such that:* $\begin{cases} \xi_n[j,t] \leq \xi_m[j,t], t \in T \\ \xi_n[j,t] < \xi_m[j,t] \textit{ for at least one } t \in T \\ F(D_{j_n}) \geq p \end{cases}$.

It is known [12, 28] that there exists at least one p-efficient demand trajectory for any discrete probability distribution regardless of the value of p. The ready rate p-efficient demand trajectories are $|T|$-dimensional unknown vectors that must be found prior to the optimization process. They can be efficiently identified with a forward enumerative algorithm [26]. Clearly, the ready rate CSL is attained if the cumulative supply $z[j,0] + \omega[j,t]$ satisfies all the conditions defined by at least one of the p-efficient demand trajectories. Therefore, we replace the joint probabilistic constraints (5) by disjunctive ones.

Theorem 1 *Any joint probabilistic constraint (5) enforcing the ready rate cycle service level can be substituted by the disjunctive constraint*

$$\bigvee_{t \in T} \left(z[j,0] + \omega[j,t] \geq \xi_1[j,t] \quad \wedge \ \ldots \ \wedge \quad z[j,0] + \omega[j,t] \geq \xi_{\left|\mathbf{S_j^{(p)}}\right|}[j,t] \right) \tag{7}$$

where $\wedge$ and $\vee$ respectively denote the conjunction and the disjunction symbols, and $\mathbf{S_j^{(p)}}$ denotes the set of service level (p) efficient demand trajectories for distributor j.

The constraint above imposes that the cumulative supply $z[j,0] + \omega[j,t]$ satisfies all ($\bigwedge_{t \in T}$) conditions imposed by at least ($\vee$) one of the p-efficient demand trajectories and transforms (6) into a disjunctive mixed-integer problem. It is well known that any disjunctive constraint can be reformulated by a set of linear constraints involving binary variables. Thus, we replace constraint (7) by the set of constraints

$$z[j,0] + \omega[j,t] \geq \beta_m[j] \cdot \xi_m[j,t], \ t \in T \tag{8}$$

$$\sum_{m=1}^{\left|\mathbf{S_j^{(p)}}\right|} \beta_m[j] \geq 1 \tag{9}$$

$$\beta_m[j] \in \{0,1\}, m = 1, \ldots, \left|\mathbf{S_j^{(p)}}\right| \tag{10}$$

which define the same feasible region. We denote by $\beta_m[j]$ a binary variable taking value 1 if the cumulative supply is large enough to cover the cumulative demand quantities $\xi_m[j,t]$ characterizing the m^{th} demand trajectory and 0 otherwise.

The substitution of (8)-(10) for (5) in the stochastic integer problem (6) transforms this latter into a deterministic MIP problem. Since the number of supply chain p-efficient demand trajectories can be extremely large, the reliance on a standard branch-and-cut algorithm could be insufficient for problems of moderate to large size. A specialized, congestion-relief column generation algorithm can be efficiently used to solve the above large-dimensional MIP (see [26, 28]). The algorithm involves the alternate optimization of a master and an auxiliary problems, which respectively consist in optimizing the production-inventory-distribution scheme for a given p-efficient demand trajectory and the selection of an alternative one that reduces the risk of congestion.

3.1.2 Intersection of Events Model

The stochastic problem (6) is particularly complex, since it involves multi-dimensional joint probability distributions. Instead, in this section, we attempt to solve (6) after having replaced the constraints (5) that contain a multi-dimensional probability distribution with an expression involving the probability distribution functions of uni-dimensional random variables.

We introduce the intersection of events model, proposed by Prékopa [35, 36] and based on the inclusion-exclusion principle, to derive an approximation of problem (6). This involves the constraint substitution presented in Theorem 2 (whose proof is given in Appendix). We denote by $p[j,t]$ the stagewise service level *attained* at distributor j and time t.

Theorem 2 *The set of constraints*

$$P(\omega[j,t] + z[j,0] \geq \xi[j,t]) \geq p[j,t],\ t \in T,\ j \in J \tag{11}$$

$$\sum_{t \in T} (1 - p[j,t]) \leq 1 - p[j], \qquad\qquad j \in J \tag{12}$$

ensures that the joint probabilistic constraint (5) enforcing a ready rate cycle service level holds.

A few comments are in order. First, the individual probabilistic constraints (11) can be replaced by their deterministic equivalent

$$\omega[j,t] + z[j,0] \geq F_t^{-1}(p[j,t]),\ j \in J, t \in T\ , \tag{13}$$

where the notation $F_t^{-1}(p[j,t]) = \min\left\{\upsilon\ :\ F_t(\upsilon) \geq p[j,t]\right\}$ denotes the $(p[j,t])$-quantile of the cumulative probability distribution of the cumulative demand of distributor j up to time t. Thus, the intersection of events bounding model allows the derivation of a ready rate sufficient demand trajectory D_{j_m} whose components $\xi_m[j,t] = F_t^{-1}(p[j,t]), t \in T$ are such that:

$$D_{j_m} = \left[F_1^{-1}(p[j,1]), \ldots, F_t^{-1}(p[j,t]), \ldots, F_{|T|}^{-1}(p[j,|T|]) : \sum_{t=1}^{|T|} (1 - p[j,t]) \leq 1 - p[j] \right]\ . \tag{14}$$

Second, constraints (12) can only be satisfied if $p[j,t] \geq p[j],\ t \in T,\ j \in J$. Clearly, the intersection of events model implies the replacement of the $|J|$ joint probabilistic constraints (5) in (6) by the $|J| \cdot (|T| + 1)$ deterministic linear constraints (12) and (13), and is an approximation of the stochastic problem (6). Indeed, the substitution of (12) and (13) for (5) shrinks the feasible region and generates higher costs for the supply chain: the demand trajectories are service level sufficient but not efficient. We note that, in the integer problem resulting from the above substitution, $p[j,t]$ are non-negative decision variables representing the *attained stagewise* service level, and $p[j]$ are parameters representing the *prescribed cycle* service level. The constraints $1 \geq p[j,t] \geq p[j]$ are induced by (12) and can be added in the formulation.

3.1.3 Robust Model

Postulating that the solution of an optimization problem of form (6) is very computationally intensive, it is proposed in [6] to simplify it by defining $p[j,t]$ (which are decision variables in (6)) as parameters, setting them equal to:

$$p[j,t] = p[j] + \frac{1-p[j]}{|T|},\ t \in T, j \in J\ . \tag{15}$$

This leads to the definition of a unique ready rate sufficient demand trajectory D_{j_m} with components

$$D^{(p)}_{j_m} = \left[F_1^{-1}\left(p[j] + \frac{1-p[j]}{|T|}\right), \ldots, F_{|T|}^{-1}\left(p[j] + \frac{1-p[j]}{|T|}\right)\right]\ . \tag{16}$$

It allows the replacement of (5) by the constraints

$$\omega[j,t] + z[j,0] \geq F_t^{-1}(p[j,t]),\ j \in J, t \in T \quad .$$

The a priori setting for the value of $p[j,t]$ provides a simpler optimization problem, but does not offer any guarantee of optimality. Indeed, as the intersection of events approach, the robust one generates an approximate formulation of problem (6) and guarantees that constraints (12) and (13) hold, which also implies the satisfaction of (5). While the robust modeling approach defines $p[j,t]$ as parameters to which a specific value is assigned and accepts a unique ready rate sufficient trajectory (16), the intersection of events approach defines $p[j,t]$ as decision variables and accepts a finite number (14) of service level efficient trajectories (the robust modeling efficient trajectory (16) is one of them). Hence, the robust approach shrinks the feasible region more than the intersection of events approach does.

3.2 Magnitude of Stockout

In this section, we consider the *fill rate* and the *conditional expected stockout* CSLs that both limit the magnitude of the stockout. They are quantity-oriented performance measures requesting that the proportion or the quantity of demand satisfied without delay from the stock on hand is above a certain value.

3.2.1 Fill Rate Service Level

The *fill rate* service level p', with $p' \in [0,1]$ and usually close to 1, is a major performance indicator [23, 37] that requires the expected fraction of product demand that cannot be met from on hand inventory to be lower than or equal to a certain proportion $(1-p')$. The complement $1-p'$ of p' is called the *unfill rate*.

We calculate the expected shortfall as follows:

$$E\left[\xi[j,t] - (\omega[j,t] + z[j,0])\right]^+ \ ,$$

where $z[j,0]$ is the known initial inventory level at j. Normalizing the expected shortfall with respect to the cumulative demand $\xi[j,t]$, we obtain

$$E\left[\frac{\xi[j,t] - (\omega[j,t] + z[j,0])}{\xi[j,t]}\right]^+ \ . \tag{17}$$

Note that the expression above represents the fraction of expected shortfall at time t and not the expected shortfall until time t. The value of (17) could be equal to 0 at $t, t = 2, \ldots, |T|$ even if a shortfall ($\xi[j,t'] > z[j,0] + \omega[j,t']$) happened at a previous period $t', t' < t$. Thus, in order to account for the stockout that happened at each period across the planning horizon, the fill rate cycle service level p' imposes that the sum of the expected fractions of stockout is upper bounded by $(1-p')$. It follows that the demand trajectory $D_{j_m} = [\xi_m[j,1], \ldots, \xi_m[j,|T|]]$ can only be fill rate (p') sufficient if the constraint below holds:

$$\sum_{t \in T} E\left[\frac{\xi[j,t] - \xi_m[j,t])}{\xi[j,t]}\right]^+ \leq 1 - p'[j] \; . \tag{18}$$

Therefore, when a fill rate CSL is pursued, we can replace the service level constraint (3) by (18) and

$$\omega[j,t] + z[j,0] \geq \xi_m[j,t], \; j \in J, t \in T \; . \tag{19}$$

The constraint (18), in which each component $E\left[\frac{\xi[j,t]-\xi_m[j,t]}{\xi[j,t]}\right]^+$ is larger than or equal to 0, ensures that the expected amount by which each inequality $\frac{\xi[j,t]-\xi_m[j,t]}{\xi[j,t]} \leq 0, t \in T$ is violated, is limited from above by $(1-p'[j])$. Hence, (18) can be replaced by the set of constraints

$$\begin{array}{lr} E\left[\frac{\xi[j,t] - \xi_m[j,t]}{\xi[j,t]}\right]^+ \leq 1 - p'[j,t] & j \in J, t \in T \\ \sum_{t \in T}(1 - p'[j,t]) \leq 1 - p'[j] & j \in J \\ 0 \leq p'[j,t] \leq p'[j] & j \in J, t \in T \\ \xi_m[j,t] \geq 0 & j \in J, t \in T \end{array} \; . \tag{20}$$

The optimal solution of the problem (1)-(4), where (19) and (20) are substituted for (3), defines the fill rate efficient demand trajectory which represents the minimal quantities $\xi_m[j,t], t \in T$ of demand to be satisfied in order to reach the prescribed fill rate CSL. The solution of this optimization problem will greatly benefit from Theorem 3 (see the Appendix for its proof).

Theorem 3 *If the univariate random vector ξ is discretely distributed, taking a finite number of possible values, then the function*

$$E\left[\frac{\xi - H\omega}{\xi}\right]^+$$

is piecewise linear and convex in $\mathcal{R}$, with H denoting the technology matrix.

Theorem 3 shows that every function $E\left[\frac{\xi[j,t]-\xi_m[j,t]}{\xi[j,t]}\right]^+$ is piecewise linear and convex. It is linear on the successive intervals $[\xi^{l-1}[j,t], \xi^l[j,t]], l = 1, \ldots, \ell^t$, where $\xi^l[j,t]$ denotes the l^{th} level that $\xi[j,t]$ can take at t, and is equal to $\sum_{l=1}^{\ell^t} \mathbf{p}^l \left[\frac{\xi^l[j,t]-\xi_m[j,t]}{\xi^l[j,t]}\right]^+$.

3.2.2 Conditional Expected Stockout Service Level

The *conditional expected stockout* p'' service level [1] ascertains the level of supply that is necessary to ensure that, when a stockout occurs, the expected quantity of missing products is below a predefined value. It bounds the conditional expected magnitude of product in stockout and is modeled by using conditional expectation constraints. In the stagewise case, which assumes that each period is considered independently of the others, the conditional expected amount

$$E\big[\xi[j,t]-(\omega[j,t]+z[j,0])\mid \xi[j,t]>\omega[j,t]+z[j,0]\big]\leq s^{p''}[j,t]\ ,\ j\in J, t\in T \tag{21}$$

is constrained to be below a prescribed quantity $s^{p''}[j,t]$ that is here defined in such a way that the quantity of conditional expected stockout does not exceed $p''\%$ of the demand received until time t:

$$s^{p''}[j,t]=\bar{\xi}[j,t]-F_t^{-1}(p''[j,t])\ ,\ t\in T, j\in J\ .$$

The notation $\bar{\xi}[j,t]$ indicates the maximum level of the cumulative demand until t and $F_t^{-1}(p''[j,t])$ is the $p''[j,t]$-quantile of the cumulative random demand at time t.

As for the fill rate service level, the value of the left-hand side in (21) could be equal to 0 at the last period $|T|$, while being strictly positive at earlier periods. Hence, the conditional expected stockout cycle service level is modeled by the constraint

$$\sum_{t\in T} E\left[\ (\xi[j,t]-(\omega[j,t]+z[j,0])\mid \xi[j,t]>\omega[j,t]+z[j,0])\right]\leq s^{p''}[j]\ ,\ j\in J\ ,\quad \text{where}$$

$$s^{p''}[j]=\bar{\xi}[j,|T|]-F_{|T|}^{-1}(p''[j,|T|]),\ j\in J\ .$$

We define $s^{p''}[j]$ as the difference between the maximum possible demand $\bar{\xi}[j,|T|]$ at j over the entire horizon and the p''-quantile $F_{|T|}^{-1}(p''[j,|T|])$ of the probability distribution of the demand at the end of the planning horizon.

Thus, $D_{j_m}=[\xi_m[j,1],\ldots,\xi_m[j,|T|]]$ is conditional expected stockout sufficient if the constraint

$$\sum_{t\in T}\left(E\left[\xi[j,t]-\xi_m[j,t]\mid \xi[j,t]>\xi_m[j,t]\right]\right)\leq s^{p''}[j]\ ,\ j\in J\ ,$$

where each term $E\left[\xi[j,t]-\xi_m[j,t]\mid \xi[j,t]>\xi_m[j,t]\right]$ is at least equal to 0, holds. Hence, the substitution of the set of constraints

$$\begin{aligned}
&\omega[j,t]+z[j,0]\geq \xi_m[j,t] && j\in J, t\in T\\
&E\left[\xi[j,t]-\xi_m[j,t]\mid \xi[j,t]>\xi_m[j,t]\right]\leq s^{p''}[j,t] && j\in J, t\in T\\
&\sum_{t\in T} s^{p''}[j,t]\leq s^{p''}[j] && j\in J\\
&0\leq s^{p''}[j,t]\leq s^{p''}[j] && j\in J, t\in T\\
&\xi_m[j,t]\geq 0 && j\in J, t\in T
\end{aligned} \tag{22}$$

for the constraint (3) ensures the attainment of the prescribed conditional expected stockout CSL. The optimal solution of the optimization problem (1)-(4), where (22) replaces (3), determines the conditional expected stockout efficient demand trajectory that represents the minimal amounts of demand to be satisfied for the conditional expected stockout service level constraint to hold.

4. Computational Study

The computational study has the following objectives: *(i)* to study the overall applicability and computational tractability of the models and the solution approach developed in this paper; *(ii)* to compare the reliability level that a supply chain can reach through the enforcement of cycle versus stagewise service levels; *(iii)* to compare the requirements imposed by a CSL limiting the probability of a stockout versus one limiting the magnitude of a stockout; *(iv)* to verify the applicability of the solution method to construct a plan that satisfies the conditions imposed by two CSLs: one limiting the probability of a stockout and one limiting the magnitude of a stockout; *(v)* to appraise the degree of conservativeness of the three models proposed for the ready rate CSL.

The computational study is divided into two main parts. First, the above questions will be investigated in the context of a real-life problem faced by a three-stage supply chain selling various forms of calcium chloryde throughout North America. In the second part of the study, in order to evaluate the overall applicability of the proposed approach, we apply it to solve more complex and more general planning models in which the constraints and assumptions specific to the supply chain studied in the first part of the computational section are removed.

The problems are modelled with the AMPL mathematical programming language and solved with the 11.1 CPLEX and the open-source Bonmin [3] solvers. Each problem instance is solved on a `64-bit Dell Precision T5400 Workstation` with `Quad Core Xeon Processor X5460 3.16GHz CPU`, and `4X2GB` of `RAM`. When optimality cannot be proven, the optimization process is stopped after one hour of CPU time.

4.1 Real-Life Problem

We describe below the real supply chain to which we first apply our approach. A more detailed description can be found in [28].

The supply chain operates in North America and is one of the five largest worldwide producers of soda ash and calcium chloryde. It is a three-stage supply chain with one supplier in Michigan, two manufacturers in Michigan and Ontario, and thirteen distributors in harbour cities such as Montreal, Quebec, Cleveland, Oswego, etc. It generates approximatively $300 million of revenue, about $100 million of which stemming from the calcium chloryde market. Very large and heterogeneous tank ships and barges are used to transport products over the Great Lakes between supply chain entities. Large ships are very expensive to operate; transportation costs represent about 50% of the total product costs. Contracting out transportation capacity with external logistics service providers is possible. The raw material is brine and the products are several forms (liquid, flake, pellet) of calcium chloryde. The demand for calcium chloryde is non-stationary and very seasonal - the product is used, for example, for motorways maintenance (de-icing of roads, etc.). Demand shortages lead to customer dissatisfaction that could be disastrous in a market with a few very large distributors. Shortage management is the key supply chain performance driver and provides the supply chain an incentive to construct replenishment plans that satisfy the conditions dictated by a stockout-related CSL.

The supply chain seeks to minimize the sum of its inventory, production, and distribution costs over a one-year planning horizon decomposed into monthly time-periods ($|T| = 12$), without violating any constraints. In order to do that, the supply chain must take optimal decisions pertaining to the *production* (production levels of raw material, semi-finished and finished products at each facility and period), *distribution* (selection of carriers to be used, scheduling and routing of each carrier in the transportation fleet: number of shipments from any supplier or manufacturer facility to any distributor or manufacturer at each period, cumulative supply: amount of product delivered to each facility at each period; periods at which the carriers owned by the company are maintained), and *inventory* (inventory levels at each facility and period) functions.

The supply chain must satisfy the following standard constraints in effect for most supply chains. The production and inventory capacities are limited at each supply chain facility (suppliers, manufacturers, distributors). The sustainability of the planning strategy requires that, at each facility, the inventory levels at the last period of the planning horizon are at least equal to the initial ones. The flow balance constraints set the ending (i.e., at the end of a period t) inventory levels equal to the initial ones (i.e., at the start of the planning horizon) increased by the cumulative (up to time t) supply minus the cumulative demand.

We provide below the formulation of the constraints involving integer decision variables as well as those that are specific to the studied chemical supply chain. The specifics of these constraints pertain to the fact that the supply chain uses a maritime distribution network. The following set notations are used in the paper: V is the set of transportation carriers, $V' \subset V$ is the set of carriers owned by the supply chain, I is the set of production facilities, $K = I \cup J$ is the set of supply chain nodes, and $K(t)$ is the set of supply chain nodes that are not accessible at time t.

The constraints

$$\sum_{i \in I} \sum_{k \in K} b[i,k,v] \cdot x[i,k,v,t] \leq a[v,t] \quad v \in V, t \in T \tag{23}$$

$$x[i,k,v,t] \in \mathcal{Z}_+ \quad i \in I, k \in K, v \in V, t \in T, \tag{24}$$

represent the *limited time availability* $a[v,t]$ of carriers at each period t, and account for the total lead time $b[i,k,v]$ to deliver from facility i to k with carrier v. The total lead time is defined as the sum of the loading, unloading, delivery, and backhaul times. The decision variable $x[i,k,v,t]$ represents the number of shipments between i and k at time t with carrier v and is defined (24) as a positive general integer variable. The maintenance of the carriers owned by the supply chain is enforced by:

$$x[i,k,v,t] \leq M \cdot \delta[v,t] \quad i \in I, k \in K, v \in V', t \in T \tag{25}$$

$$\sum_{t \in T} \delta[v,t] = |T| - 1 \quad v \in V' \tag{26}$$

$$\delta[v,t] \in \{0,1\} \quad v \in V', t \in T \tag{27}$$

Constraints (25) and (26) are the *carrier maintenance* constraints imposing that each carrier belonging to the supply chain is not used in at least one period, allowing for its maintenance during (part of) that time. We denote by M the maximum number of shipments between i and k that can be done at t with v, and we introduce a binary variable $\delta[v,t]$ (27) equal to 1 if the carrier v is chartered at t, and equal to 0 otherwise. The *distribution time-window* constraints

$$x[i,k,v,t] = 0, \ i \in I, t \in T, k \in K(t), v \in V$$

account for the fact that some facilities k are not accessible at some time t, due, for example, to bad weather conditions. For instance, no transportation to Cleveland is allowed during the winter, since the chemical product is then likely to freeze. The constraints

$$q[i,k,v,t] = c[v] \cdot x[i,k,v,t], \ i \in I, k \in K, v \in V, t \in T \ , \tag{28}$$

where $q[i,k,v,t]$ denotes the amount of products delivered to k at t with v leaving from i, and $c[v]$ is the loading capacity of v, enforce a *full-load* and direct (i.e., a single loading and discharging location) shipment policy. This distribution policy results from the fact that the demand is much larger than the ships' maximal loading capacity, and from the very high cost of operating a ship. Such a policy appears to be frequently used in maritime distribution [22]. For an in-depth discussion of the advantages of resorting to direct shipments, we refer the reader to [17].

The objective function is linear and minimizes the sum of the distribution, production, and holding costs incurred at each supply chain node k over the entire planning horizon. The production costs are computed by multiplying the production levels of semi-finished products and end-products by their unit production costs. The inventory costs are proportional to the ending stock levels. Most often, the transportation costs are piecewise linear and defined as the sum of a fixed cost incurred if a carrier is chartered (28) and a variable cost depending on the quantity of products loaded on the carrier and the lead time between the delivering and receiving facilities. The full-shipment policy used in this supply chain, which requires the delivery of a quantity equal to the capacity of the carrier used, allows, however, the modeling of the distribution costs $\sum\limits_{i\in I}\sum\limits_{k\in K}\sum\limits_{t\in T}\sum\limits_{v\in V} g[i,k,v]\cdot x[i,k,v,t]$, where $g[i,k,v]$ is the total transportation cost incurred by chartering carrier v and by using it to transport a product quantity $c[v]$ from node i to node k at time t, as a function of the continuous decision variables $q[i,k,v,t]$:

$$\sum_{i\in I}\sum_{k\in K}\sum_{t\in T}\sum_{v\in V} \frac{g[i,k,v]}{c[v]}\cdot q[i,k,v,t]\ , \tag{29}$$

which takes value 0 if there is no shipment between i and k with v at t. The full-load shipment policy implies that the ratios $\frac{q[i,k,v,t]}{c[v]}$ and $\frac{g[i,k,v]}{c[v]}$ respectively indicate the number of shipments and the unit transportation cost with carrier v leaving from i and heading to k at period t.

We use historical data to derive two probability distributions for the random demand. The first (resp., second) probability distribution is symmetric (resp., positively skewed) and the demand $d[j,t]$ can take $\ell = 5$ (resp., 10) different levels at each time-period. For these two probability distributions, we consider three values 0.9, 0.95, and 0.97, for each service level. The following statistics give an idea of the size of the problem: we have about 850 continuous variables, 500 general integer variables, 500 binary integer variables, and 2200 constraints. The exact numbers vary with the type of CSL, the prescribed value of the service level, and the modeling approach. We complement the optimization solvers by using a family of binary-integer cover inequalities [27] that are very efficient for dealing with the distribution constraints containing integer variables (23).

4.2 Computational Tractability

4.2.1 Probability of Stockout

In this section, we study the performance of our solution method to solve the integer optimization problems associated with the three modeling approaches for the ready rate CSL. As compared to the robust model (Section 3.1.3), the one based on the intersection of events approach (Section 3.1.2) comprises $|J|$ additional continuous decision variables and linear constraints, while the p-efficiency model (Section 3.1.1) contains $|J|$ additional covering constraints and $\sum\limits_{j\in J}\left|S_j^{(p)}\right|$ additional binary decision variables. Table 1 shows that the number of p-efficient demand trajectories (and therefore the number of additional binary variables) is a decreasing function of the value of p, and generally increases with the number of different demand realizations. It is worth noting that the value of $\left|S_j^{(p)}\right|$

does not increase exponentially with ℓ. The very different number of p-efficient demand trajectories per distributor is due to the non-stationarity (i.e., the specifics) of the distributor demands.

	p	CLE	DAR	LIT	MON	MOR	NEW	OSH	OSW	OWE	QUE	SEP	STE	THU
	0.9	22	29	34	121	51	44	134	48	3	83	8	17	164
$\left\|S_j^{(p)}\right\|$	0.95	13	23	26	86	35	33	104	32	1	68	6	13	130
$\ell = 5$	0.97	8	14	18	40	16	8	53	16	1	38	4	3	61
	0.9	30	32	38	129	50	45	142	68	8	92	9	22	162
$\left\|S_j^{(p)}\right\|$	0.95	21	24	21	99	28	20	121	52	6	58	6	10	151
$\ell = 10$	0.97	10	11	8	44	14	15	64	25	6	28	3	9	68

Table 1: Number of p-Efficient Demand Trajectories

To evaluate the computational tractability of the proposed approaches, we compute

- the integrality gap $I = \frac{UB-LB}{LB}$ for the p-efficiency model: UB is the value of the best feasible integer solution found. The value of LB is obtained by constructing and solving to optimality the convexification of the continuous relaxation of the MIP problem associated with the p-efficiency approach:

$$\begin{aligned}
&\min f(x) \\
&\text{subject to } Ax \geq b \\
&\qquad z[j,0] + \omega[j,t] \geq \sum_{m=1}^{\left|S_j^{(p)}\right|} \lambda_m[j] \cdot \xi_m[j,t] && j \in J, t \in T \\
&\qquad \sum_{m=1}^{\left|S_j^{(p)}\right|} \lambda_m[j] = 1 && j \in J \\
&\qquad \lambda_m[j] \geq 0 && j \in J, m = 1, ..., \left|S_j^{(p)}\right| \\
&\qquad x \in \mathcal{R}_+
\end{aligned}$$

- the optimality gap $O = \frac{UB-BB}{BB}$ for the robust and intersection of events models: BB is the best bound identified by the solver.

For each combination of p and ℓ (Table 2), we have generated 10 problem instances and we have solved them with each modeling approach. Entries in Table 2 report the value of the average optimality and integrality gaps for each combination of p and ℓ and for each modeling approach, as well as the average time (CPU seconds) needed to find the best solution throughout the one hour allowed.

The computational tractability of the solution approach, when applied to the robust and the intersection of events models, is attested by the very small values of the optimality gaps for each set of problem instances. The same conclusion can be extended to the p-efficiency approach for which the integrality gap is always very low. We do not report the value of the optimality gap for the p-efficiency model, since the solution approach is a column generation algorithm. At each iteration, a particular column, i.e., p-efficient demand trajectory D_{j_m}, is considered, and a best bound BB can only be computed with respect to that specific demand trajectory. The computational times for the intersection of events and robust models are roughly the same and are lower than those for the p-efficiency model.

		Intersection of Events		Robust Model			p-Efficiency	
		$\ell = 5$	$\ell = 10$	$\ell = 5$	$\ell = 10$		$\ell = 5$	$\ell = 10$
p = 0.9	O	0.38%	0.49%	0.29%	0.39%	I	2.65%	2.92%
	Time	235	251	209	198	Time	561	489
p = 0.95	O	0.20%	0.34%	0.24%	0.33%	I	2.81%	3.10%
	Time	196	259	191	209	Time	444	648
p = 0.97	O	0.33%	0.30%	0.27%	0.46%	I	2.67%	2.89%
	Time	226	184	178	165	Time	651	429

Table 2: Solution Quality and Computing Time with Intersection of Events, Robust and p-Efficiency Approaches

4.2.2 Magnitude of Stockout

We proceed to the same analysis for the two CSLs that limit the magnitude of the stockout. Entries in Table 3 report the average optimality gap (for each combination of service level magnitude and ℓ) corresponding to the best replenishment plan satisfying a fill rate (left side) and conditional expected stockout (right side) CSL. The computational tractability of the proposed approach is attested by the very small value of the average optimality gap and the very limited average computational time for each set of problem instances. As above, we have generated and solved 60 problem instances (i.e., 10 for each combination of values taken by p' or p'' and ℓ).

Fill Rate					Conditional Expected Stockout				
	ℓ = 5	Time	ℓ = 10	Time		ℓ = 5	Time	ℓ = 10	Time
p' = 90%	0.84%	154	0.93%	154	p'' = 90%	0.77%	161	0.76%	142
p' = 95%	0.39%	180	0.73%	181	p'' = 95%	0.65%	176	0.85%	187
p' = 97%	0.54%	172	0.66%	184	p'' = 97%	0.63%	198	0.71%	174

Table 3: Average Optimality Gap and Computing Time with Fill Rate and Conditional Expected Stockout Cycle Service Level

4.3 Stockout Occurrence and Magnitude with Stagewise and Cycle Service Levels

4.3.1 Probability of Stockout

In this section, we investigate and compare the reliability level, defined as the probability of not having any stockout across the planning horizon, with the three following approaches:

- P-EFF: cycle service level modeled using the p-efficiency concept;
- IPC: enforcement of a stagewise service level $p[j,t]$ at each period $t \in T$ considered independently of the other periods in the horizon. This is modeled with individual probabilistic constraints in which the random independent right-hand side is the periodic demand $d[j,t]$:
  $$P(z[j,t-1] + m[j,t] \geq d[j,t]) \geq p_{j,t}, \ t \in T, j \in J \quad .$$
  The notations $z[j,t-1]$ and $m[j,t]$ respectively denote the inventory levels at period $(t-1)$ at facility j and the supply provided to j at the start of period t. Replacing $d[j,t]$ by its unidimensional, and therefore unique, p-quantile, the problem above turns into a deterministic MIP;

- EXP: replacement of the random demand $\xi[j,t]$ by its expected value $E[\xi[j,t]]$. Thus, we obtain a deterministic optimization problem in which the constraint $z[j,0]+\omega[j,t] \geq E[\xi[j,t]],\ j \in J, t \in T$ is to be satisfied instead of the probabilistic constraint (5).

First, for each problem setting (i.e., value given to ℓ and $p[j]$), we construct the optimal replenishment plan associated with each of the above approaches. Second, we generate 100 demand trajectories (i.e., scenarios) for the considered distribution of the random demand and calculate for how many of those a shortage happens with the optimal replenishment plan. We do that count for each of the three approaches, which provides us with an estimate of the horizon-wide probability of a stockout with the optimal plan of the three approaches. Figure 1 displays the results of the simulation study for the problem instances in which $\ell = 5$ and $p[j] = 0.95 = p[j,t], j \in J, t \in T$.

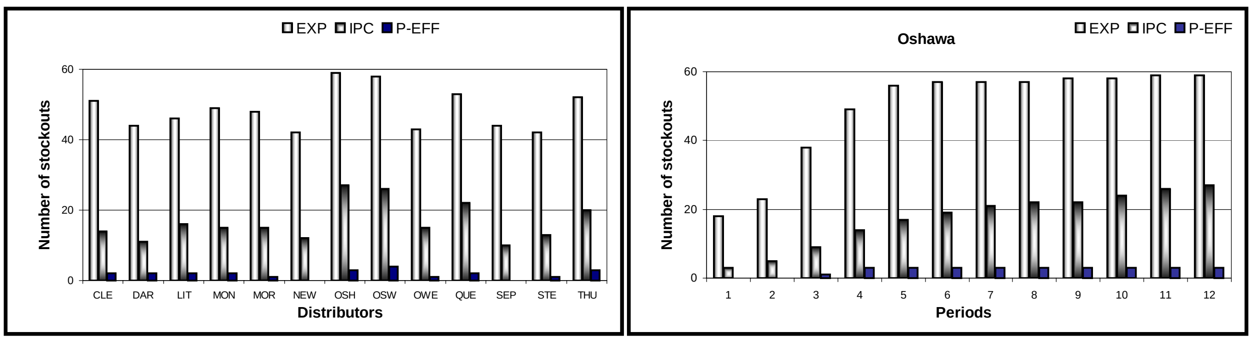

Figure 1: Comparison of Modeling Approaches for the Ready Rate Service Level

The chart on the left side of Figure 1 shows that the supply chain experiences at least one stockout over the entire planning horizon in many scenarios (at least 42%, at most for 59%) with the optimal plan of the EXP approach. The IPC approach enforcing stagewise service levels does not reflect the desired safety requirements either. It guarantees that, at each period, the probability of negative inventory level is below 5%. Yet, the probability of having a shortout at least once in the planning horizon varies between 10% and 27%, and remains much higher than the probability (5%) allowed by the prescribed ready rate CSL. The plan constructed using the p-efficiency approach satisfies the conditions of the desired CSL. The same comments apply for all problem instances.

The chart on the right side of Figure 1 displays the cumulative (up to period t) number of stockouts experienced by the Oshawa-based distributor for the three evaluated approaches. We can see that the cumulative number of stockouts increases fast (almost reaching its maximum) at periods 4, 5, and 6 when the demand reaches its peak and the risk of congestion is the highest. This simulation study illustrates the non-suitability of enforcing stagewise service levels if one seeks to have a small probability of stockout over the entire planning horizon.

4.3.2 Magnitude of Stockout

In this section, we compare the conditional expected stockout (resp., fill rate) cycle and stagewise service levels. The fill rate stagewise service level constraints are formulated as

$$\sum_{l}^{\ell^t} \mathbf{p}^l \left[\frac{\xi^l[j,t] - (\omega[j,t] + z[j,0])}{\xi^l[j,t]} \right]^+ \leq 1 - p'[j,t],\ j \in J, t \in T\ ,$$

while the formulation of the conditional expected stockout stagewise service level constraint is given in (21). We construct the optimal replenishment plan satisfying a conditional expected stockout (resp., fill rate) cycle (resp., stagewise) service level ($\ell = 5, p''[j,t] = p'[j,t] = 0.95, j \in J, t \in T$). We then calculate in how many of 100 simulated demand trajectories the optimal cycle and stagewise replenishment plans violate the requirements imposed by conditional expected stockout (resp., fill rate) CSL ($p''[j] = p'[j] = 0.95, j \in J$). The left (resp., right) graph in Figure 2 shows the number of demand trajectories, among the 100 simulated ones, for which the quantity (resp., fraction) of product stockout exceeds $s^{(0.95)}[j]$ (resp., 5%).

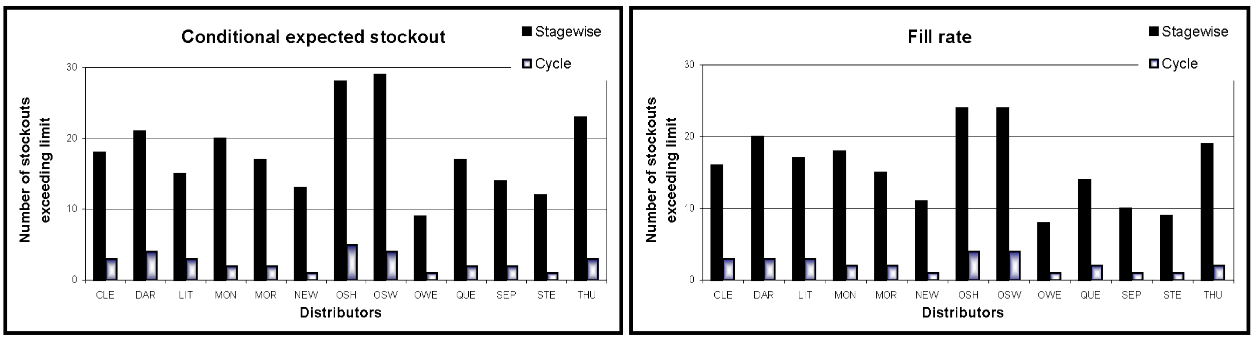

Figure 2: Comparative Study of Stagewise and Cycle Service Levels

Figure 2 shows that the supply chain experiences a conditional expected amount of stockout (resp., proportion of product stockout) larger than $s^{(p'')}[j]$ (resp., $p'[j]$) in many scenarios with the replenishment plan associated with the stagewise approach of the conditional expected stockout (resp., fill rate) service level. The stagewise approach clearly does not permit the attainment of the targeted CSL requirements.

4.4 Comparison of Cycle Service Level Requirements

In this section, we assess the requirements of the ready rate and fill rate CSLs, and we calculate:

- the ready rate CSL $\overline{p}$ attained by the optimal replenishment plan enforcing a fill rate CSL p' equal to 0.95 (3rd row in Table 4);
- the fill rate CSL $\overline{p'}$ attained by the optimal replenishment plan enforcing a ready rate CSL p equal to 0.95 (5th row in Table 4).

Table 4 reports the detailed results for the case when the random demand can take 5 different levels, and provides a clear empirical confirmation that the ready rate CSL is more demanding than the fill rate CSL. The enforcement of a ready rate CSL $p = 95\%$ guarantees a fill rate CSL ($\overline{p'}$) higher than 99.9%, while a fill rate CSL $p' = 95\%$ guarantees a very low, ranging between [33.67%, 74.64%], ready rate CSL. Similar conclusions apply for other values of p and p' and for the other probability distribution.

4.5 Model Conservativeness for Ready Rate Cycle Service Level

In Figure 3, we plot the *stochastically efficient frontier* of the ready rate CSL with each modeling approach. A replenishment plan is said to be *stochastically efficient* if it is the least costly one that allows the attainment of a given ready rate CSL p (with a certain modeling approach). We define

	CLE	DAR	LIT	MON	MOR	NEW	OSH	OSW	OWE	QUE	SEP	STE	THU
$p' = 0.95$													
$\overline{p}$	33.67%	53.54%	34.35%	49.02%	61.22%	40.91%	39.89%	37.68%	74.64%	63.50%	44.12%	53.50%	65.73%
$p = 0.95$													
$\overline{p'}$	99.94%	99.91%	99.91%	99.92%	99.91%	99.93%	99.92%	99.94%	99.92%	99.92%	99.97%	99.92%	99.91%

Table 4: Comparison between Fill Rate and Ready Rate Cycle Service Levels

the stochastically efficient frontier as the collection of stochastically efficient plans. In Figure 3, the numbers associated with costs (7-digit numbers) are normalized (due to confidentiality requirements) with respect to the most expensive plan constructed.

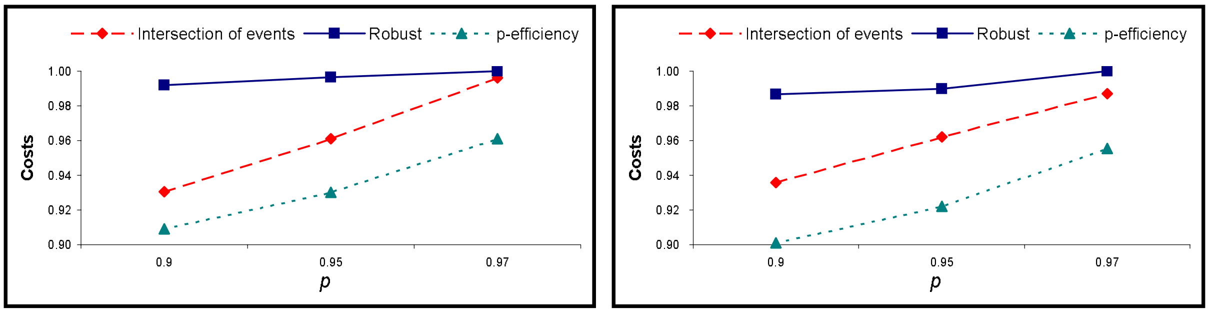

Figure 3: Efficient Frontier

Figure 3 (right: $\ell = 5$, left $\ell = 10$) highlights that the p-efficiency approach results, for each value of p, in a significantly less costly solution than the ones found with the robust and intersection of events approaches. Figure 3 shows that the total costs obtained with

- the intersection of events approach for $p = 0.90$ (resp., 0.95) are larger than those obtained with the p-efficiency approach for $p = 0.95$ (resp., 0.97);
- the robust approach for $p = 0.90$ (thus, also $p = 0.95, 0.97$) are larger than those obtained with the p-efficiency approach for $p = 0.97$.

The reason for the above cost results is that the intersection of events approach is more constraining and gives an upper bound, not always very tight, on the optimal solution of (6). Indeed, as it can be inferred from constraints (11) and (12), the higher the dimensionality (value of $|T|$) of the random variable, the looser the bound provided by the intersection of events model. This observation evidently carries over to the robust approach, since this latter is even more conservative than the intersection of events model to approximate (6). Figure 3 shows that the total costs obtained with the robust approach for $p = 0.90$ are almost as high as those obtained with the intersection of events approach for $p = 0.97$. The rationale for using the robust formulation approach instead of the intersection of events approach is that one obtains a less complex optimization problem: by setting $p[j,t] = p[j] + \frac{1-p[j]}{|T|}$ (15), one actually trades off solution quality for computational tractability. In this study, this trade-off is not beneficial, since, for each instance, the best solution found with the intersection of events approach is better than the one found with the robust approach, and both solutions are obtained in similar computational times. Finally, Figure 1 highlights that the total costs with the robust model remain stable, and are almost invariant to the value of the enforced CSL.

The following analysis is another illustration of the conservativeness of the intersection of events and the robust approaches and of their potential relevance for risk-averse decision-makers. For each problem instance (i.e., for each value of $p = 0.9, 0.95, 0.97$ and $\ell = 5, 10$), we construct the best possible plan using the three modeling approaches for the ready rate CSL.

Considering a sample of 100 simulated demand trajectories, we compare for each problem instance and modeling approach:

- the *enforced* ready rate CSL $p[j]$;
- the *obtained* ready rate CSL $\overline{p}[j]$, which is the probability of not having any shortage across the planning horizon by implementing the optimal plan built for the enforced service level $p[j]$;
- the *true* ready rate CSL $\widetilde{p}[j]$, which is the fraction of the 100 simulated demand trajectories for which there is no stockout if one implements the optimal plan.

Figure 4 displays the enforced, obtained, and true service levels for each solution approach when the random stagewise demand can take $\ell = 5$ demand levels at each period. Results are almost identical for $\ell = 10$.

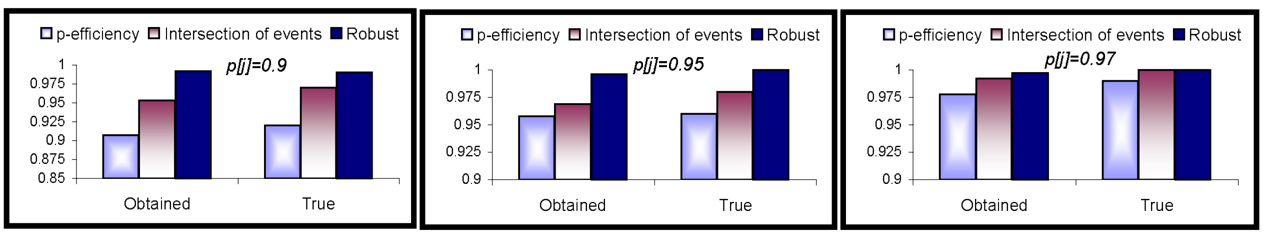

Figure 4: Comparison of Enforced, Obtained and True Cycle Service Levels

The true and obtained service levels with the robust and the intersection of events models are significantly higher than the enforced ones, since the two approaches are approximations of (6), requiring the satisfaction of stricter requirements than those of the ready rate CSL. The robust approach results in a service level larger than the one obtained with the intersection of events model, confirming the higher conservativeness of the robust approach. The fact that the obtained service level with the p-efficiency approach is higher than the enforced one can, at first sight, appear surprising, but it is the consequence of the full-load shipment policy that results in the delivery of quantities in excess to those needed for the enforced service level.

As reported in the thorough review of the maritime transportation literature [10, 38], most proposed maritime distribution models are deterministic [11, 14, 15, 34], although ship scheduling carries much uncertainty. Our paper is, to the best of our knowledge, the first one to propose models and a solution method that allow for the construction of a stochastic, integrated [41], and multi-period planning problem with maritime distribution policy.

4.6 Generalization to Standard and Larger Multi-Stage Supply Chains

This section is intended to verify the computational tractability and applicability of our approach to a wide range of multi-stage supply chains. This will be accomplished by *(i)* relaxing the assumptions (full-load shipments, cost function) that are particular to the chemical supply chain, and *(ii)* increasing the dimensionality of the problem. This latter goal will involve the consideration of a more complex supply chain network, an extended fleet of distribution carriers, and a larger number of demand realizations at each period of the horizon.

The full-load shipment constraint (28), particular to the chemical supply chain analyzed so far, has a strong impact on the formulation of the planning optimization problem. First, the decision to use a carrier v at time t for transporting products from i to k fully determines the quantity of products (i.e., equal to the maximal loading capacity $c[v]$ of the carrier v) delivered with this shipment. Second, the transportation cost can be formulated as a continuous function (29). Third, the reason why we distinguish the "obtained" and "enforced" service levels resides in the fixed quantity of products delivered in each shipment. This results in the delivery of product quantities that exceed those that would be needed to exactly reach the enforced service level.

The removal of the full-load shipment restriction and the allowance for partial-load shipments requires the reformulation of the transportation constraint as:

$$q[i,k,v,t] \leq c[v] \cdot x[i,k,v,t], \; i \in I, k \in K, v \in V, t \in T \; , \tag{30}$$

As a consequence, the objective function is modified and transportation costs are modeled as a piecewise linear function [30]:

$$\sum_{i\in I}\sum_{k\in K}\sum_{v\in V}\sum_{t\in T} h[v] \cdot x[i,k,v,t] + \sum_{i\in I}\sum_{k\in K}\sum_{v\in V}\sum_{t\in T} r[i,k,v] \cdot q[i,k,v,t] \; , \tag{31}$$

where $h[v]$ is the fixed cost incurred for chartering carrier v total and $r[i,k,v]$ is the cost for transporting one product form i to k with carrier v. The values of $h[v]$ and $r[i,k,v]$ are such that $h[v] + c[v] \cdot r[i,k,v] = g[i,k,v]$.

To assess the extendibility of the proposed approach to larger and standard supply chain networks, we have generated 120 problem instances in which *(i)* we have removed the full-load shipment constraint and instead use formulations (30) and (31) for the distribution constraint and the distribution cost function, and *(ii)* the supply chain network is of larger size and complexity (i.e., numbers $|Q|$, $|J|$, and $|L|$ of suppliers, manufacturers, and distributors, the number $|V|$ of carriers, and the number ℓ of realizations that the demand $d[j,t]$ can take at each period t and distributor j). Tables 5 and 6 characterize the problem instances. All the problem instances were solved with each of the approaches proposed in this paper.

Table 5 reports the quality of the obtained solution as well as the time required to reach the best feasible solution within one hour of computing time. This information is provided for the three models proposed for the ready rate CSL. We have generated and solved 10 problem instances for each combination of value taken by p and ℓ. It appears clearly that our approach results in very low optimality and integrality gaps for networks of larger size. We observe that the computing time increases with the complexity of the network, but does so at a very reasonable rhythm.

Table 6 provides the same information as Table 5 for the two service levels limiting the magnitude of the stockout. As for the ready rate CSL, the computational results show the applicability (i.e., quality of the solution obtained in reasonable computational times) of the proposed method for general supply chain networks of larger dimensionality.

$\lvert Q\rvert$	$\lvert L\rvert$	$\lvert J\rvert$	$\lvert V\rvert$	ℓ	p	Intersection of Events		Robust Model		p-Efficiency	
						O	Time	O	Time	I	Time
2	4	15	10	15	0.9	0.20%	320	0.24%	287	2.21%	625
2	4	15	10	20	0.9	0.38%	306	0.39%	251	2.48%	689
2	4	15	10	15	0.95	0.14%	289	0.21%	254	1.98%	574
2	4	15	10	20	0.95	0.19%	387	0.23%	326	2.96%	722
4	4	20	10	15	0.9	0.32%	342	0.26%	302	1.79%	759
4	4	20	10	20	0.9	0.29%	364	0.32%	254	2.41%	826
4	4	20	10	15	0.95	0.31%	421	0.41%	328	2.54%	847
4	4	20	10	20	0.95	0.22%	389	0.29%	341	3.03%	956
4	8	20	15	15	0.9	0.32%	542	0.35%	490	2.23%	1025
4	8	20	15	20	0.9	0.36%	521	0.27%	396	3.41%	1109
4	8	20	15	15	0.95	0.24%	625	0.34%	489	3.62%	1274
4	8	20	15	20	0.95	0.39%	642	0.29%	525	2.10%	1326

Table 5: Generalization - Part I: Probability of Stockout

$\lvert Q\rvert$	$\lvert L\rvert$	$\lvert J\rvert$	$\lvert V\rvert$	ℓ	p', p''	Fill Rate		Conditional Expected Stockout	
						O	Time	O	Time
2	4	15	10	15	0.9	0.30%	282	0.41%	305
2	4	15	10	20	0.9	0.46%	250	0.48%	329
2	4	15	10	15	0.95	0.18%	274	0.79%	287
2	4	15	10	20	0.95	0.28%	235	0.39%	341
4	4	20	10	15	0.9	0.45%	296	0.54%	369
4	4	20	10	20	0.9	0.63%	347	0.29%	396
4	4	20	10	15	0.95	0.31%	325	0.64%	421
4	4	20	10	20	0.95	0.58%	365	0.59%	362
4	8	20	15	15	0.9	0.61%	487	0.73%	589
4	8	20	15	20	0.9	0.49%	536	0.82%	642
4	8	20	15	15	0.95	0.38%	479	0.41%	521
4	8	20	15	20	0.95	0.46%	548	0.58%	539

Table 6: Generalization - Part II: Magnitude of Stockout

4.7 Combination of Cycle Service Levels

In order to simultaneously ensure the attainment of the fill rate and ready rate CSLs (i.e., thus restricting infeasibilities to occur with no more than a probability $(1-p)$ and in no more than a prescribed expected proportion $(1-p')$), we solve the following problem

$$
\begin{aligned}
&\min c^T x \\
\text{subject to } & Ax \geq b \\
& P(z[j,0] + \omega[j,t] \geq \xi[j,t],\ t \in T) \geq p[j] && j \in J \\
& \sum_{t \in T} E\left[\frac{\xi[j,t] - z[j,0] - \omega[j,t]}{\xi[j,t]}\right]^+ \leq 1 - p'[j] && j \in J \\
& x = [x'\ x''] \in \mathcal{R}_+ \times \mathcal{Z}_+
\end{aligned}
$$

using the solution approaches discussed previously. For each problem instance, the optimal solution turns out to be the same as the one obtained with the ready rate CSL. Indeed, the fill rate service level constraints is redundant, imposing easier requirements than those of the joint probabilistic constraints representing the ready rate CSL.

5. Conclusion

Demand shortage results in very damaging customer dissatisfaction and loss of goodwill, and is a key performance driver for supply chains. In this paper, we consider multi-stage supply chains operating in uncertain environments and whose goal is to construct an integrated replenishment plan that satisfies stockout-related cycle service level requirements. The concept of cycle refers to the duration of the entire planning horizon composed of a finite number of interdependent time-periods. A CSL requires the likelihood (resp., magnitude) of a shortage happening at any point during the planning horizon to be lower than a small prescribed probability (resp., quantity). Cycle service levels are very demanding, and are increasingly needed by companies operating in global and highly competitive environments.

We use a discrete-time static stochastic programming framework to derive new integrated planning models enforcing ready rate, fill rate, and conditional expected stockout CSLs. We propose a solution approach that can be uniformly applied to each type of CSL. The solution approach rests on the concepts of service level sufficient (resp., efficient) demand trajectory which is a multi-dimensional vector representing a set of sufficient (resp., minimal) demand requirements that must be satisfied by the supply chain in order to attain the targeted CSL.

We proceed to an extended computational study to evaluate the proposed models and solution approach. The first part of the computational study is based on a real-life problem faced by a major North American chemical supply chain using a maritime transportation network. It is shown that: *(i)* the proposed approach is applicable to handle the very specific requirements of this supply chain to build replenishment plans satisfying CSL requirements; *(ii)* the inadequacy of enforcing stagewise service levels for supply chains aiming at satisfying more demanding, horizon-wide non-stockout performance metrics; *(iii)* the three modeling approaches proposed for the ready rate CSL result in true service levels of different magnitudes. We show that the p-efficiency model provides a true service level which is the closest to the enforced one, and permits substantial cost savings. We show that the intersection of events and robust modeling approaches are approximations of the model enforcing a ready rate CSL. In particular, we show that the robust modeling approach is very conservative.

The second part of the study highlights the overall applicability and high computational performance of the proposed approach. These conclusions are based on the results (low optimality/integrality gaps, limited computing time) obtained by applying our approach to very general and larger multi-stage supply chain networks. We consider more complex networks (i.e., higher number of nodes at each level of the supply chain), a larger transportation network, and a larger number of levels that the random demand can take at each distributor and at any period in the planning horizon.

The point above underlines another key aspect of the present study, namely its wide applicability. The modeling and solution approach is appropriate and applicable for most multi-tiered communicative, coordinated, and collaborative supply chains [29]. The scope of our approach is further widened by the fact that the random demand is not restricted by any independence assumption. The proposed approach can be used for time-dependent, stationary, or non-stationary (i.e., seasonal) demand.

Note that the proposed approach is general enough to cope with other cost functions. For example, consider that *(i)* the production costs at a facility i equal to $o[i] + w[i] \cdot u[i,t], t \in T$, where $o[i]$ is the fixed set-up cost and $h[i]$ is the constant production marginal cost, and assume further that the facility cannot be activated if it does not produce at least a prescribed minimal quantity $\underline{u}[i]$; *(ii)* a production facility i can benefit from economies of scale and the production costs are accordingly defined as:

$$c[i] = \begin{cases} o''[i] + w''[i] \cdot u[i,t], & \text{if} \quad u[i,t] > u'[i], \\ o''[i] + w'[i] \cdot u[i,t], & \text{if} \quad 0 < u[i,t] \leq u'[i], \quad t \in T \ , \\ 0, & \text{if} \quad u[i,t] = 0, \end{cases}$$

with $w''[i] < w'[i]$. In both settings, which have been frequently studied in the literature [19], the cost function takes the form of a concave and piecewise linear function and can be tackled with our approach. We also show that the proposed approach can be used to simultaneously enforce several types (i.e., limiting the probability or the magnitude of stockout) of CLSs without jeopardizing its computational tractability.

We assume in this paper that the probability distribution of the demand is discrete. In case of a continuous one, the planning models take the form of convex mixed-integer stochastic problems (for most continuous probability distributions). In this case, the main challenge of the solution method is the calculation of the cumulative distribution function and of its gradient values which involve numerical integration or simulation in high dimensional spaces. Further research endeavors will concern the construction of integrated plans enforcing CSL when the source of uncertainty originates from another source (i.e., lead times, exchange rate for international supply chains, etc.) than the demand.

Acknowledgments

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Appendix

Proof of Theorem 2

Proof: We have that:

$$\begin{aligned} & P\left(\xi[j,t] \leq \omega[j,t] + z[j,0], t \in T\right) & j \in J \\ &= 1 - P\left(\bigcup_{t \in T} \left(\xi[j,t] > \omega[j,t] + z[j,0]\right)\right) & j \in J \\ &\geq 1 - \sum_{t \in T} P\big(\xi[j,t] > \omega[j,t] + z[j,0]\big) & j \in J \ . \end{aligned} \tag{32}$$

From (32), constraints (5) in (6) are always satisfied if the inequality

$$1 - \sum_{t \in T} \left(1 - P\left(\omega[j,t] + z[j,0] \geq \xi[j,t]\right)\right) \geq p[j], \ j \in J$$

holds for each j. This requires that constraints

$$\begin{aligned} & 1 - \sum_{t \in T} \left(1 - P\left(\omega[j,t] + z[j,0] \geq \xi[j,t]\right)\right) \geq 1 - \sum_{t \in T} (1 - p[j,t]) & j \in J \\ & 1 - \sum_{t \in T} (1 - p[j,t]) \geq p[j] & j \in J \end{aligned}$$

hold jointly, which is guaranteed if

$$\begin{cases} P\left(\omega[j,t] + z[j,0] \geq \xi[j,t]\right) \geq p[j,t] & j \in J, t \in T \\ 1 - p[j] \geq \sum\limits_{t \in T} (1 - p[j,t]) & j \in J \end{cases} . \qquad \square$$

Proof of Theorem 3

Proof: *i)* Piecewise linearity. Let $\mathbf{p}^l$ be the probability for ξ taking value ξ^l, $I \subset \{1, 2, \ldots, \ell\}$, and

$$Y(I) = \left\{ \omega \,\middle|\, \frac{\xi^l - H\omega}{\xi^l} \geq 0,\ l \in I,\ \frac{\xi^{l'} - H\omega}{\xi^{l'}} < 0,\ l' \notin I \right\} .$$

It can be seen that

$$E\left[\frac{\xi - H\omega}{\xi}\right]^+ = \sum_{l \in I} \mathbf{p}^\mathbf{l} \, \frac{\xi^l - H\omega}{\xi^l}$$

if $\omega \in cl\, Y(I)$, $cl\, Y(I)$ denoting the closure of $Y(I)$. The above function is linear on each set $cl\, Y(I)$, and, since $cl\, Y(I)$ is a convex polyhedron and $\bigcup\limits_{I \subset \{1,\ldots,\ell\}} cl\, Y(I) = \mathcal{R}$, is piecewise linear in $\mathcal{R}$.

ii) Convexity. Since the one-dimensional function $[z]^+$ is convex, it follows that

$$\left[\frac{\xi^l - H(\lambda\omega_1 + (1-\lambda)\omega_2)}{\xi^l}\right]^+ \leq \lambda \left[\frac{\xi^l - H\omega_1}{\xi^l}\right]^+ + (1-\lambda)\left[\frac{\xi^l - H\omega_2}{\xi^l}\right]^+ \tag{33}$$

for any $\omega_1, \omega_2 \in \mathcal{R},\ 0 \leq \lambda \leq 1$. Multiplying (33) by $\mathbf{p}^l$ and summing the inequalities, we get

$$E\left[\frac{\xi - H(\lambda\omega_1 + (1-\lambda)\omega_2)}{\xi}\right]^+ \leq \lambda E\left[\frac{\xi - H\omega_1}{\xi}\right]^+ + (1-\lambda) E\left[\frac{\xi - H\omega_2}{\xi}\right]^+ . \qquad \square$$

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