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The Institute for Integrating Statistics in Decision Sciences

Technical Report TR-2010-2

February 10, 2010

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Summary. In this paper we present a Bayesian framework for modeling uncertainty about a population's health state preferences. Such a framework is motivated by the need to analyze preference-based measurement data that arise from evaluation of health states by a sample of individuals. The Bayesian framework leads to population utility estimation and health policy evaluation by introducing a probabilistic interpretation of the multiattribute utility theory (MAUT) models used in health economics. In so doing, our approach combines ideas from the MAUT approach of Keeney and Raiffa [10] and Bayesian view point to provide an alternate method of modeling preferences.

Key words: Bayesian analysis; Health economics; Multiattribute utility; Monte Carlo

1 Introduction

Preference based measurement of health (PBMH) methods have been developed to be used in economic evaluation of health policies. Use of preference based measures requires quantification of health state preferences by a group of individuals. This preference data is used as a sample to develop an aggregate measure for the population preferences. A recent overview of PBMH methods can be found in Brazier and Roberts [4]. The methods that quantify preference based measurement of health (PBMH) are referred to as the health related quality of life measures (HRQoL). The methods include

- Health Utilities Index (HUI)of Torrance et. al [17]
- Quality of Well-Being (QWB) scale of Kaplan et. al [9]
- Short Form (SF-6D) survey of Brazier, Roberts and Deverill [2].

These measures are used to quantify a population's preferences over health states that have multiple dimensions. Thus, the measures are based on multi attribute evaluation of states using preference weights and scores. They provide a single index number for each health state. Typically an index value "1" denotes perfect health and "0" denotes death. In the health economics literature, these index values are referred to as utility. Elicitation of utility requires sophisticated procedures based on standard gambles as discussed in Brazier and Deverill [1]. A more simplified approach for obtaining preference measures is to ask respondents to assign values to health states directly and have the analyst convert these to utilities.

Preference based measures such as QWB and SF-6D use what is referred to as the *composite approach* for estimation of the multi attribute utility function for the health states. The composite approach involves direct elicitation of utility of multidimensional health states and requires more health states than that can be evaluated by a single respondent; see Brazier [3]. Regression type models are used to extrapolate the values of health states that are not included in the survey. An alternative for estimation of the utilities for the health states is the *decomposed approach* employed in the HUI. This method uses the MAUT framework of Keeney and Raiffa [10] and determines a functional form for the multi attribute utility function of health states. The decomposed approach is based on simplifying assumptions such as *preferential independence* and *utility independence*; see for example, Keeney [11]. The approach yields simpler forms of utility functions and substantially reduces the valuation effort by decomposing the problem into one-dimensional elicitation problems. In addition to providing evaluations of all possible health states, the decomposed approach is also flexible in modeling interactions using multiplicative utility functions. As noted by [3], this is unlike the composite approach where there is no standard method for determining the states required to estimate a model with interaction terms. Hazen [8] describes how the additive or multiplicative decomposition within QALYs can be constructed using the independence concepts and discusses how they relate to HUI.

Both the composite and decomposed approaches provide us with a sample of health state valuation data, that is, with health state utilities from a sample of individuals. The objective is to estimate the health state utilities of the population based on this sample and use the estimated population utility function to evaluate different health policy alternatives. Statistical methods have been considered by earlier researchers such as Dolan [5] and Brazier et al.[2]. In general these approaches employed linear models with normally distributed error terms. As pointed out by Brazier [3], these models, that used data from the composite approach, "have estimated crude summary terms for interactions" and have required range of transformations to deal with highly skewed data.

Most of the earlier models and associated statistical methods have used classical sampling theoretic approaches. More recently, Bayesian approaches have been considered in the health economics literature. For example, nonparametric Bayesian models are proposed by Kharroubi, O'Hagan and Brazier [12] and by Kharroubi et. al [13] for HRQoL estimation of a population. Standard gambles are used by the authors to elicit utilities of health states from the sample of individuals. Although a sampling method is used by authors to avoid an individual going large number of gambles, the proposed composite approach still requires large number of evaluations of health states. In view of this, following Musal et. al [15], we consider a parametric Bayesian approach and use a decomposed model in our framework. Our framework consists of modeling attribute utilities, modeling attribute weights and using a multiattribute model for aggregation.

In Section 2 we introduce a model for single attribute utilities. Using a decomposed approach and an additive utility function, modeling of attribute weights is presented in Section 3. We discuss Bayesian aggregation of attribute utilities and weights as well as probabilistic evaluation of health state preferences in Section 4. Concluding remarks are given in Section 5.

2 Bayesian Modeling of Attribute Utilities

Assume that preference ordering of the $K_c + 1$ levels with respect to each attribute $c = 1, \ldots, C$,

$$X_{c,1} \prec X_{c,2} \prec \cdots \prec X_{c,K_c} \prec X_{c,K_c+1} \tag{1}$$

is identical for the population, where $X_{c,j}$ denotes the jth level of a single attribute c. We are interested in making inference about the unknown population utilities

$$u(X_{c,2}) < \cdots < u(X_{c,K_c}), \tag{2}$$

where $u(X_{c,1}) = 0$ and $u(X_{c,K_c+1}) = 1$. We may have a prior opinion on these values and we are interested in updating this prior opinion based on the sample utility measurements on the N individuals. In general $u(X^i_{c,j}) = u^i_{c,j}$ is the utility declared by the $i - th$ individual for attribute c at level j.

We focus on a single criterion and to reflect the ordering that applies to the population we assume that for all individuals

$$0 < u_2 < u_3 < \cdots < u_K < 1. \tag{3}$$

It is desirable to have a probability model for utility vectors $\mathbf{u}^i = (u^i_2, \ldots, u^i_K)$ which is consistent with the ordering and flexible enough to reflect the diminishing utility scenario encountered in many applications. As suggested in Musal et. al [15], the *ordered Dirichlet* model can provide that kind of flexibility in describing different utility scenarios. The ordered Dirichlet model has been previously used in reliability growth modeling by Mazzuchi and Soyer [14] and Erkanli, Mazzuchi and Soyer [7] who pointed out such properties with respect to behavior of reliabilities over time. The ordered Dirichlet model for the utility vector $\mathbf{u} = (u_2, u_3, \ldots, u_K)$ is given by

$$p(\mathbf{u} \mid \beta, \alpha) = \frac{\Gamma(\beta)}{\prod_{j=2}^{K+1} \Gamma(\beta\alpha_j)} \prod_{j=2}^{K+1} (u_j - u_{j-1})^{\beta\alpha_j - 1}, \tag{4}$$

where $u_1 = 0$ and $u_{K+1} = 1$ and the distribution is defined over the simplex (3). The model parameters are β and α such that $\beta > 0, \alpha_j > 0$ and $\sum_{j=2}^{K+1} \alpha_j = 1$.

In the ordered Dirichlet model (4), the marginals are beta densities denoted as

$$(u_j \mid \beta, \alpha) \sim Beta\Big(\beta\alpha^*_j, \beta(1 - \alpha^*_j)\Big) \tag{5}$$

for $j = 2, \ldots K$, where $\alpha^*_j = \sum_{k=2}^{j} \alpha_k$, $E[u_j \mid \beta, \alpha] = \alpha^*_j$ and β is the precision parameter where lower values are associated with a more diffused distribution of the utility level. As pointed out by van Dorpe, Mazzuchi and Soyer [6], for $i < j$, the model implies that

$$(u_j - u_i) \mid \beta, \alpha \sim Beta\Big(\beta(\alpha^*_j - \alpha^*_i), \beta(1 - \alpha^*_j + \alpha^*_i)\Big). \tag{6}$$

Thus, the changes in the adjacent utilities $(u_j - u_{j-1})$, for $j = 2, \ldots, K + 1$, follow a beta distribution where the mean is given by

$$E[u_j - u_{j-1} \mid \beta, \alpha] = (\alpha^*_j - \alpha^*_{j-1}) = \alpha_j. \tag{7}$$

It follows from the above that, α_j can be interpreted as the expected increase in utility as a result of going from attribute level X_{j-1} to attribute level X_j whereas α_j^* is the expected utility at attribute level X_j. We note that α_j^* is increasing with j, implying that for the population we expect utility is an increasing function of the attribute when high values of the attribute are desirable. If $E[u_j - u_{j-1} \mid \beta, \alpha] = \alpha_j$ is a decreasing sequence in j, then we expect that the marginal utility is diminishing as the attribute level gets larger. In this case, we will have $E[u_j \mid \beta, \alpha] = \alpha_j^*$ is discrete concave in j. Thus, the model is attractive in that it allows for incorporation of different prior information about expected behavior of utilities into the analysis. Such prior beliefs can be used in specification of the prior distribution of the parameters α and β.

In the Bayesian paradigm uncertainty about all unknown quantities are described probabilistically. Thus, completion of Bayesian modeling of attribute utilities requires us to specify the prior distribution of α and β. We will denote the prior by $p(\beta, \alpha)$. It is not unreasonable to assume that α and β are independent a priori. Since β is the precision parameter, a reasonable prior distribution for β is the gamma distribution. As pointed out by Musal et. al [15], in specifying the prior distribution of α, one can use the properties of the ordered Dirichlet model discussed above. More specifically, if we have prior beliefs about the monotonocity of α_j's, then a distribution which reflects that belief will be the appropriate choice. However, in modeling health state preferences, one of the objectives is to infer about such monotonic behavior and thus, a prior which does not force an ordering of α_j's is more desirable. Since the ordered Dirichlet model (4) requires that $\sum_{j=2}^{K+1} \alpha_j = 1$, a natural prior for α is the Dirichlet distribution

$$p(\alpha \mid b, \mathbf{a}) = \frac{\Gamma(b)}{\prod_{j=2}^{K+1} \Gamma(ba_j)} \prod_{j=2}^{K+1} (\alpha_j)^{ab_j - 1}, \tag{8}$$

where $a_j > 0$ and $b > 0$ are specified parameters such that $\sum_{j=2}^{K+1} a_j = 1$. Note that a_j's are the expected values of α_j's and b is the precision parameter of the model.

2.1 Bayesian Analysis of Utilities

Our objective is to describe uncertainty about the population utility $\mathbf{u} = (u_2, \ldots, u_K)$ based on the information provided by the sample of N utility vectors

$$\mathbf{u}^i = (u_2^i, u_3^i, \ldots, u_K^i), i = 1, \ldots, N,$$

where $\mathbf{u}^i$'s are assumed to follow the ordered Dirichlet model (4). Thus, given sample utilities $\mathbf{u}^N = (\mathbf{u}^1, \mathbf{u}^2, \ldots, \mathbf{u}^N)$ from the N individuals, we are interested in the posterior predictive distribution $p(\mathbf{u}|\mathbf{u}^N)$. The distribution $p(\mathbf{u}|\mathbf{u}^N)$ describes our inferences about population utilities for a given attribute.

The posterior predictive distribution is obtained via the calculus of probability as

$$p(\mathbf{u}|\mathbf{u}^N) = \int_{\beta,\alpha} p(\mathbf{u}|\mathbf{u}^N, \beta, \alpha) p(\beta, \alpha|\mathbf{u}^N) d\beta d\alpha. \tag{9}$$

Using the conditional independence of $\mathbf{u}$ and $\mathbf{u}^N$, given β and α, (9) reduces to

$$p(\mathbf{u}|\mathbf{u}^N) = \int_{\beta,\alpha} p(\mathbf{u}|\beta, \alpha) p(\beta, \alpha|\mathbf{u}^N) d\beta d\alpha, \tag{10}$$

where $p(\mathbf{u}|\beta, \alpha)$ is given by (4) and $p(\beta, \alpha|\mathbf{u}^N)$ is the posterior distribution of the parameters of the ordered Dirichlet model. Once sample utilities $\mathbf{u}^N$ is available, uncertainty about α and β is revised via Bayes' rule

$$p(\beta, \alpha|\mathbf{u}^N) \propto L(\beta, \alpha; \mathbf{u}^N)p(\beta, \alpha), \tag{11}$$

where $L(\beta, \alpha; \mathbf{u}^N)$ is the likelihood function based on the ordered Dirichlet distribution (4). More specifically, we have

$$L(\beta, \alpha; \mathbf{u}^N) = \prod_{i=1}^{N} \left[\frac{\Gamma(\beta)}{\prod_{j=2}^{K+1} \Gamma(\beta\alpha_j)} (u_j^i - u_{j-1}^i)^{\beta\alpha_j - 1} \right]. \tag{12}$$

The posterior distribution $p(\beta, \alpha|\mathbf{u}^N)$ can not be obtained analytically for any choice of the prior distributions discussed earlier. However, we can draw samples from the posterior distribution using Markov chain Monte Carlo (MCMC) methods; see Musal et. al [15] for details. Since $p(\beta, \alpha|\mathbf{u}^N)$ is not analytically available, evaluation of the posterior predictive distribution (10) requires use of Monte Carlo methods. Given samples $\left(\beta^{(s)}, \alpha^{(s)}\right)_{s=1}^{S}$ from the posterior distribution $p(\beta, \alpha|\mathbf{u}^N)$, we can approximate (10) via the Monte Carlo average

$$p(\mathbf{u}|\mathbf{u}^N) \approx \frac{1}{S} \sum_{s=1}^{S} p(\mathbf{u}|\beta^{(s)}, \alpha^{(s)}). \tag{13}$$

By using the predictive distribution we can make probability statements about utilities at each of the attribute levels and approximate the population's *expected utility function* by plotting the $E(u_j|\mathbf{u}^N)$ versus the attribute level X_j's. We can also provide posterior probability bounds for the utilities.

3 Modeling Attribute Weights

The development in Section 2 is presented for a single attribute, that is, for attribute c, with $K_c + 1$ levels and observed utility vectors $\mathbf{u}_c^i = (u_{c,2}^i, \ldots, u_{c,K_c}^i)$ for individuals $i = 1, \ldots, N$. In general for specifying the ordered Dirichlet model for the multiattribute problem, all model parameters are indexed by c, that is, we actually have (β_c, α_c) with prior $p(\beta_c,\alpha_c)$.

If we have mutual utility independence of the attributes (see for example, [10]), then the above development can be easily extended to C attributes. In this case the parameters (β_c, α_c) are assumed to be independent for $c = 1, \ldots, C$ and thus the approach presented in Section 2 yields independent posterior predictive distributions $p(\mathbf{u}_c|\mathbf{u}_c^N)$ for $c = 1, \ldots, C$. The mutual utility independence of the attributes justifies the use of multiplicative multiattribute utility model as considered by Torrance, Boyle and Hardwood [16] for describing society's preference for health states. Furthermore, if *additive utility independence* (see Keeney [11]) can be justified, then the multiattribute utility function can be written as

$$u(X_1, X_2, \ldots, X_C) = \sum_{c=1}^{C} w_c u(X_c), \tag{14}$$

where w_c's are weights representing the relative importance of the attributes such that $0 < w_c < 1$ and $\sum_{c=1}^{C} w_c = 1$.

In the multiattribute utility (MAU) function (14), $u(X_c)$'s are unknown utilities whose posterior distributions are available to us via using Monte Carlo based methods. The weight vector $\mathbf{w} = (w_1, , w_2, \ldots, w_C)$ is also an unknown quantity. Thus, from a Bayesian perspective for given levels of $X_1, X_2, \ldots, X_C$, uncertainty about the MAU needs to be described probabilistically. Such a development requires us to consider a probability model for the attribute weight vector $\mathbf{w}$. The model needs to be consistent with the requirement that $\sum_{c=1}^{C} w_c = 1$ and that the weights are negatively correlated. An appropriate probability model for this case is the Dirichlet distribution

$$p(\mathbf{w} \mid \kappa, \gamma) = \frac{\Gamma(\kappa)}{\prod_{c=1}^{C} \Gamma(\kappa\gamma_c)} \prod_{c=1}^{C} (w_c)^{\kappa\gamma_c - 1}, \tag{15}$$

where $\gamma = (\gamma_1, \ldots, \gamma_C)$. It is well known that all the marginal distributions are Beta densities, that is,

$$(w_c \mid \kappa, \gamma) \sim Beta\Big(\kappa\gamma_c, \kappa(\gamma_0 - \gamma_c)\Big), \tag{16}$$

where $\gamma_0 = \sum_{c=1}^{C} \gamma_c = 1$. Prior distributions of the unknown parameters κ and γ can be specified to reflect these properties. For example, the prior for γ can be specified as a Dirichlet distribution. Since γ_c's represent expected attribute weights in the model, the prior parameters can be chosen to reflect our best guesses about attribute weights in the population. As before, a gamma density is a reasonable prior for the precision parameter κ.

Health state preference data typically include attribute weights $\mathbf{w}^i = (w_1^i, w_2^i, \ldots, w_C^i)$, $i = 1, \ldots, N$, elicited from the sample of N individuals (see for example, [15]). As before, we treat the N weight vectors as the samples from the Dirichlet distribution in (15). Thus, it is possible to develop Bayesian machinery to revise our uncertainty based on such data.

If we specify, $p(\kappa, \gamma)$, as the prior distribution for κ and γ then given sample weights, $\mathbf{w}^N$, from N individuls, the posterior distribution is obtained via

$$p(\kappa, \gamma | \mathbf{w}^N) \propto L(\kappa, \gamma; \mathbf{w}^N) p(\kappa, \gamma), \tag{17}$$

where the likelihood function, $L(\kappa, \gamma; \mathbf{w}^N)$ is based on the Dirichlet model (15). The posterior distribution (17) can not be analytically obtained for any reasonable choice of the prior $p(\kappa, \gamma)$. As before, MCMC methods can be used to draw samples from (17). As in the case of attribute utilities of Section 2, the objective is to make inference about the attribute weight vector of the population. Thus, we are interested in obtaining the posterior predictive distribution

$$p(\mathbf{w} | \mathbf{w}^N) = \int p(\mathbf{w} | \kappa, \gamma) p(\kappa, \gamma | \mathbf{w}^N) d\kappa d\gamma. \tag{18}$$

The integral in (18) can not be obtained analytically, but given samples $\big(\kappa^{(s)}, \gamma^{(s)}\big)_{s=1}^{S}$ from the posterior distribution (17) we can approximate it via the Monte Carlo average

$$p(\mathbf{w} | \mathbf{w}^N) \approx \frac{1}{S} \sum_{s=1}^{S} p(\mathbf{w} | \kappa^{(s)}, \gamma^{(s)}), \tag{19}$$

where $p(\mathbf{w}|\kappa^{(s)}, \gamma^{(s)})$ is the Dirichlet density. Note that once we simulate the weight vectors $\mathbf{w}^{(s)}$, for $s = 1, \ldots, S$, from the Dirichlet distribution, then we can make probability statements about attribute weights such as $Pr(w_i > w_j|\mathbf{w}^N)$. In other words, we can infer probabilistically if certain attributes are more important than the others for the population.

4 Bayesian Evaluation of Health States

In Sections 2 and 3, we have presented a Bayesian framework for modeling attribute utilities and weights. In so doing, we have discussed how to obtain two sets of posterior samples, that is, sample of utilities for given attributes and attribute weights. Using these, we can make probability statements using the MAU function (14). Given samples from the posterior predictive distributions $p(\mathbf{u}_c|\mathbf{u}_c^N)$ for $c = 1, \ldots, C$ and $p(\mathbf{w}|\mathbf{w}^N)$, we can evaluate the population utility distribution for a specific health state.

For a specific health state, A_i with the attribute levels, $(X_{1,A_i}, ..., X_{C,A_i})$, we can obtain the probability distribution of the corresponding MAU via the Monte Carlo evaluation of $u\left(X_{1,A_i}, ..., X_{C,A_i}\right)$ using (14). For A_i, we can write

$$u\left(X_{1,A_i}, ..., X_{C,A_i}\right) = \sum_{c=1}^{C} w_c u(X_{c,A_i}), \tag{20}$$

and using the posterior samples we can obtain a histogram estimate of the posterior distribution of $u\left(X_{1,A_i}, ..., X_{C,A_i}\right)$. In a similar manner, we can make probability statements on whether health state A_i is preferred to state A_j in the population, that is,

$$Pr(A_i \succ A_j|\mathbf{u}_1^N, \ldots, \mathbf{u}_C^N, \mathbf{w}^N).$$

This probability is equivalent to

$$Pr\Big\{u\left(X_{1,A_i}, \ldots, X_{C,A_i}\right) > u\left(X_{1,A_j}, \ldots, X_{C,A_j}\right)|\mathbf{u}_1^N, \ldots, \mathbf{u}_C^N, \mathbf{w}^N\Big\} \tag{21}$$

which can be approximated using the posterior samples.

The literature on the preference based measures of health generally has considered other alternatives to the additive model. A common method of decomposition that is used to account for potential interactions in attribute utilities is the multiplicative utility model. Musal et. al [15] considered a multiplicative model and developed a Bayesian framework similar to what is presented here.

5 Concluding Remarks

In this paper we have presented a Bayesian framework for modeling uncertainty about a population's health state preferences. Our development is based on the composite approach as in Torrance, Boyle and Hardwood [16]. The Bayesian framework involves modeling both the attribute utilities and attribute weights and provides probabilistic evaluation of health state preferences. Since our focus has been on the Bayesian perspective here, computational issues involving implementation of MCMC methods for developing posterior inferences are not discussed in the paper. For this we refer the interested reader to Musal et. al [15] where such methods are applied in analysis of actual health state preference data.

It is possible to consider extensions of the models proposed here. For example, the precision parameters β_c's, in modeling utilities, can be assumed to be constant across the attributes. This can be one way to impose a dependence structure for different attribute utilities. Other possible extensions include incorporation of covariate information in the utility and attribute weight models and taking into consideration heterogeneity of individuals. Such issues have been consdered in [15].

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