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A Paired Comparison Experiment for Gathering Expert Judgment for
An Aircraft Wiring Risk Assessment

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Abstract

Wire failure in aircraft can be attributed to several factors and the assessment of the risk of wire failure is becoming an increasingly important task. This paper will discuss the results of an actual experiment to use the paired-comparison technique for expert judgment to develop a relationship for the probability of wire failure as a function of influencing factors in an aircraft environment. The reasons for using this technique are two-fold. First, the failure probability depends on many variables including wire gauge, vibration, environmental condition etc. In addition, the wire failure data is sparse and fitting this data to a complex failure function is a nontrivial task that may involve a host of assumptions that may not be provable.

. We describe a method for using actual failure data and the results from a paired comparison to populate the model parameters. In the approach, paired comparison data from select environments is used to obtain failure rate estimates for the candidate environments. Next, a functional relationship for wire failure as a function of the environments is constructed using a proportional hazards model. A regression model is fit from the failure rate estimates to the environmental variables and is used as an estimate of the failure response surface. This technique is being investigated as a means to generate failure rates for an Electrical Wiring Interconnection System Risk Assessment software tool currently being developed for the FAA Tech Center.

1. Introduction

Many systems in a modern aircraft are electrical and the trend is for the aircraft to become more and more electric. Components of these systems are often distributed in different location in the aircraft and are connected by the Electrical Wire Interconnect System (EWIS). The reliability of these systems is therefore dependent, in part, on the reliability of the EWIS, of which, wire is a major component.

Wire reliability is dependant on the inherent properties of the wire itself (insulation type, wire gauge etc), properties of the bundle of wires in which it is routed (curvature, orientation, protection , etc.), and the zonal environment (vibration, temperature, exposure to fluids etc) in which the wire is

located. Because differences in wire properties and environments can affect the reliability of the wire it would be desirable to have a wire failure rate function that took these variables into consideration. However, the number of different environments produced by different combinations of wire properties and zonal environments that realistically occur on aircraft is overwhelming. In addition, historical wire failure data is sparse thus making the estimation of such a multivariate function impossible by usual statistical techniques.

To bridge this gap of historical data, a wire failure paired comparison workshop was held using 14 experts from the aviation community. The technique of paired comparison using expert opinion was used to obtain relative failure rates of a set of candidate environments. These results were then used to obtain an expression for the wire failure rate as a function of wiring environment. Section 2 provides an overview of the procedure and Section 3 provides an overview of the paired comparison approach. The results are discussed in Section 4.

2. Modeling for Wire Failure

The purpose of this section will be to develop a theoretically sound model for wire failure. By "wire failure" we specifically refer to two modes of failure

  i.   Fail to ground - including wire to wire and wire to structure failure
  ii.   Fail to open – broken conductors

2.1 Development of a Time to Failure Distribution

We consider the development of a time to failure probability density function (pdf) for wire failure based on environmental factors. We begin by assuming the form of the pdf. The pdf for T_g and T_o, the time to wire failure for the failure modes "fail to ground" and "fail to open" respectively is assumed to be the exponential distribution given by

$$f(t_i|\lambda_i) = \lambda_i e^{-\lambda_i t_i} \tag{1}$$

where $i = g, o$ and the parameter $\lambda_i > 0$ is referred to as the failure rate for failure mode i. To completely specify the distribution, this parameter must be estimated, usually from past data. The exponential distribution has been applied successfully for years in reliability and risk analysis to model the failure behavior of electronic components [see for example, Nelson (1982) and Meeker and Escobar (1998) among many others]. Assuming that the individual failure modes behave independently (which is a common assumption unless a particular dependence model can be specified), it is well known that the time to wire failure (irregardless of failure mode), $T = \min\{T_g, T_o\}$ has an exponential pdf with failure rate $\lambda_g + \lambda_o$. Thus we may consider each failure mode separately in our analysis.

The simple form of the pdf in (1) is not flexible enough for our needs if we wish to consider environmental factors that may accelerate the failure process. Through discussion with industry experts, a list of these environmental factors and their critical values was compiled. This list is presented in Table 1 below.

Incorporating environmental factor into a time to failure pdf is a common practice in reliability and biometry. A common model for incorporating these variables is the proportional hazards model (PHM) [see for example Lawless (2003)]. The basic idea of the model is to write the failure rate as a function of the covariates, a common form being

$$\lambda = e^{\beta_0 + \beta_1 X_1 + \dots + \beta_{15} X_{13}}. \tag{2}$$

Table 1: Environmental Factors Contributing to Wire Failure

Variables	Levels			
	1	2	3	4
Wire Guage	4\0-8 awg	10-16 awg	18-22 awg	24-26 awg
Conductor Type	Aluminum	Copper	High Streng. Copper Alloy	
Splices	None	Environmental	Non-environmental	
Bundle Protection	Some Level of Protection	Not Protected (Open)	Protected Metal Conduit	
Curvature of Bundle	Low (> 10x)	High (<= 10x)		
Ops/Main Traffic	Low	Moderate	High	
Vibration	Low	Moderate	High	Extreme
Ops temp\presurization	Benign (P&T Controlled)	D1- P Contrl. but not T	D2 (P&T not controlled)	D3 (High T, P not contrl)
Exp Corrosive Fluid	No	Yes		
Exp Conducting Fluid	No	Yes		
Bundle Size	Large (> 1.25 in)	Moderate (0.5-1.25 in)	Small (0.2-0.5 in)	Very Small (< 0.2 in)
Insulation Type	Polyimide	Hybrid (PI/FP Composite)	ETFE & other FPs	
Bundle Orientation (Shock)	Horizontal/Vertical Wire	Longitudinal		

where the X_i represent the quantitative effect of covariate i and β_i represent regression parameters relating the influence of covariate i on the failure rate. For example, we rewrite (1) as

$$f(t|\beta_0, \beta_1, \ldots, \beta_{14}) = [e^{\beta_0 + \Sigma_{j=1,13} \beta_j X_j}] \exp\{ -[e^{\beta_0 + \Sigma_{j=1,13} \beta_j X_j}]t\} \tag{3}$$

and now it is the parameters $|\beta_0, \beta_1, \ldots, \beta_{13}$ that must be estimated from past data. Note that, we have suppressed the index i for the failure mode. We will continue to do so to save on notation but we emphasize here that a separate analysis for each failure mode must be conducted.

2.2 Traditional Model Inference

In order for the model in (3) to be of use, we must estimate the parameters $\beta_0, \beta_1, \ldots, \beta_{13}$ from past data. A well-known estimate of the failure rate for the exponential distribution is the ratio of the total number of observed failures to the total exposure time. This estimate assumes perfect repair of components. In order to use this estimate for each environment, we would need failure data from all possible failure environments. The total number of failure environments can be determined as the number of possible combinations of values of $X_1, \ldots, X_{13}$ or 4*3*3*….*3*2 = 995,328 potential environments. Note that some combinations of the covariates may not constitute a realistic environment. However, even removing those environments which are not possible would still leave a sizeable number of environments for which there is simply not enough failure data to support inference in this manner. Lack of data is usually the case in many risk analyses [see for example Bedford and Cooke (2001)].

An alternative procedure would be to obtain a sample number of such environments where failure data exists and use regression or standard DOE analysis to estimate the parameters. That is, suppose that we were able to identify m (>13) candidate environments where the kth environment is represented by specific values for the variables denoted by $x_{1,k}, \ldots x_{13,k}$. Given the m candidate environments where n_k failures were observed over an exposure time of $T_k^{exposure}$ for evironment k, k = 1, …, m, the failure rate estimate for environment k is $n_k/T_k^{exposure}$. Given (2), for each environment we may specify a model

$$n_k/T_k^{exposure} = \{e^{\beta_0 + \beta_1 x_{1,k} + \ldots + \beta_{15} x_{13,k}}\}\delta_k \qquad k=1, \ldots, m \tag{4}$$

where δ_k are random error terms. Assuming that these errors are independent and identically distributed lognormal random variables, we may rewrite (4) as

$$\text{Ln}\{n_k/T_k^{\text{exposure}}\}= \beta_0+\beta_1 x_{1,k}+\dots + \beta_{13} x_{13,k} + \text{ln}\{\delta_k\} \qquad k=1, \dots, m \qquad (5)$$

and can estimate the parameters $|\beta_0, \beta_1, \dots, \beta_{13}$ and test validity of our assumptions using residual analysis from standard multivariate linear regression [see for example, Draper and Smith (1998)].

Unfortunately there is also not sufficient data for this type of analysis. As is usual in risk analysis we must resort to another source of data to estimate our parameters. This source of this new form of data is expert judgment.

3. An Inference Procedure Based on Expert Judgment and Data

Expert judgment, or subjective data, has been used successfully in risk analysis for years [see for example, Cooke (1991)] and there are several techniques in practice for collecting, combining, and using expert judgment. One of these methodologies is called the Negative Exponential Life (NEL) model, which is based on a popular expert judgment elicitation method known as paired comparison [see for example Cooke (1991)]. Our approach will consist of four steps:

i. Select a number of failure environments to compare via paired comparison. For one of the environments selected there should also exist a reasonable amount of existing failure data. Conduct the paired comparison with the candidate environments. The result of the paired comparison will be a set of failure rate estimates obtained to within proportionality constant.

ii. Given the failure rate estimates obtained using i, obtain the parameters estimates of $\beta_0, \beta_1, \dots, \beta_{13}$ based on the regression analysis discussed in (5), with the failure rate estimates obtained in i substituted for the values $n_k/T_k^{\text{exposure}}$ in (5).

iii. Obtain a failure rate estimate of the form (4) from the candidate failure environment for which there exists significant exposure time and failure data.

iv. By comparing the failure rate estimate for the failure environment selected in iii using (4) to the failure rate estimate using the paired comparison and regression results in i and ii, the constant of proportionality for all failure rate estimates can be estimated.

Once the estimates for the parameters $\beta_0, \beta_1, \dots, \beta_{13}$ are obtained, the complete failure rate and corresponding pdf may be specified for any environment. The focus of this paper will be on steps i and ii.

3.1 The Paired Comparison NEL Model

Paired Comparison is general name for a technique used to combine several experts' beliefs about the relative probabilities (or rates of occurrence) of certain events. While there is a host of literature on this topic, two main models emerge as those most cited and most used, Thurstone (1927), and Bradley and Terry (1952). In these approaches, experts are asked to compare n items pairwise, indicating their preference for one or the other item. The Negative Exponential Lifetime (NEL) Model found in Cooke (1991) is an adaptation of the Bradley-Terry Model whereby experts are asked to compare n components or environments pairwise, indicating which component or environment is more likely to produce an earlier failure.

To summarize the procedure, we define the following notation. Let $E_1, \dots, E_n$ denote the failure environments whose failure rates we desire from e experts. Experts are asked a series of paired comparisons as to which environment is more severe, that is, more likely to produce a failure sooner. Let $N_r(i)$ represent the number of times that expert r ranked E_i more severe than the other environments in the comparisons. The paired comparison results yield values $N_r(1), \dots, N_r(n)$ for each expert r = 1, …, e.

There are many analyses possible based on the expert choices. The first analysis would be to see if each expert is specifying a true preference structure in his/her answers or just assigning answers in a random fashion. This can be determined by analyzing the number of circular triads in his/her comparisons. A circular triad occurs when the expert suggests, for example, that E_1 is more severe than E_2, E_2 is more severe than E_3, and E_3 is more severe than E_1, thus violating the transitivity property. When experts compare a large number of events, however, it is not surprising that a few circular triads may result. David (1963) determined that $c(r)$, the number of circular triads in expert r's preferences, is given by

$$c(r) = \frac{n(n^2-1)}{24} - \frac{1}{2}\sum_{i=1}^{n}\left(N_r(i) - \frac{1}{2}(n-1)\right)^2 . \tag{6}$$

Kendall (1962) developed tables of the probability that certain values of $c(r)$ are exceeded under the null hypothesis that the expert answered in a random fashion for n = 2, …, 10. In addition, Kendall (1962) developed the following statistic for comparing n items in a random fashion,

$$c'(r) = \frac{n(n-1)(n-2)}{(n-4)^2} + \left(\frac{8}{n-4}\right)\left[\left(\frac{1}{4}\right)\binom{n}{3} - c(r) + \frac{1}{2}\right] . \tag{7}$$

When n>7, this statistic has (approximately) a chi-squared distribution with $\dfrac{n(n-1)(n-2)}{(n-4)^2}$ degrees of freedom. This statistic can be used to test the null hypothesis that an expert answered randomly versus the alternative hypothesis that his/her answers form an actual preference structure. We may use this statistic to perform a standard one-tailed hypothesis test. If the null hypothesis for any expert cannot be rejected at the 5% level of significance, the expert should be dropped from the analysis.

In addition to the above analysis, the agreement of the experts as a group can be statistically tested. Let $N(i,j)$ denote the number of times some expert ranked E_i more severe than E_j. To test the hypothesis that all agreements of experts are due to chance, Kendall (1962) defines the *coefficient of agreement* as

$$u = \frac{2\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\binom{N(i,j)}{2}}{\binom{e}{2}\binom{n}{2}} - 1 \tag{8}$$

and tabulated distributions of $\sum_{i=1}^{n}\sum_{j=1,j\neq i}^{n}\binom{N(i,j)}{2}$ for small values of n and e under the hypothesis that all agreements of the experts are due to chance. For large values of n and e, Kendall (1962) also developed a statistic which under the null hypothesis that all agreements of experts is due to chance. These distributions can be used to test hypothesis concerning u. For large values of n and e, Kendall (1962) developed the statistic

$$u' = \frac{4\left[\sum_{i=1}^{n} \sum_{j=1,j\neq i}^{n} \binom{N(i,j)}{2} - \frac{1}{2}\binom{e}{2}\binom{n}{2}\left(\frac{e-3}{e-2}\right)\right]}{e-2}$$

(9)

which under the null hypothesis that all agreements of experts is due to chance, has (approximately) a

chi squared distribution with $\dfrac{\binom{n}{2}e(e-1)}{(e-2)^2}$ degrees of freedom.

Additionally, a measure referred to as the *Coefficient of Concordance* can be used to test the agreement of the experts. Letting $R(i,r)$ denote the rank of E_j obtained through expert r's responses, the Coefficient of Concordance is defined as

$$w = \frac{S}{\frac{1}{12}e^2(n^3 - n)}$$

(10)

where

$$S = \sum_{i=1}^{n}\left[\sum_{r=1}^{e} R(i,r) - \frac{\sum_{j=1}^{n}\sum_{r=1}^{e} R(j,r)}{n}\right]^2$$

(11)

The value w attains the value 1 for complete agreement. Tables of critical values developed for distribution of S under the null hypothesis that all agreements of experts is due to chance for $3 \leq n \leq 7$ and $3 \leq e \leq 20$ by Siegel (1956). For n>7, Siegel (1956) provides the statistic

$$w' = \frac{S}{\frac{1}{12}en(n+1)}$$

(12)

which is (approximately) Chi Squared with df=n−1. For both statistics, the one-tailed hypothesis that all agreements are due to chance should be rejected at the 5% level of significance in order for us to have confidence in the expert estimates.

After eliminating those experts who fail the hypothesis test provided by (7) and given the rejection of the hypothesis that the expert agreement is due to chance using (9) and/or (12), the estimate of the environment failure rates may be obtained to within a constant of proportionality. The NEL model uses the fact that given two environments say E_i, and E_j with respective failure rates h_i and h_j, the probability that environment E_i produces a failure before environment E_j is given by

$$r(i,j) = \frac{h_i}{h_i + h_j} \; ;$$

(13)

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an identical setup for the Bradley-Terrry Model.

Using the data obtained from the paired comparisons

$$N_1(1), \dots, N_1(n); N_2(1), \dots, N_2(n); \dots\dots; N_e(1), \dots, N_e(n)$$

denote $N(i)$ as the number of times some expert ranks E_i more severe than other environments, that is $N(i) = \sum_{r=1}^{e} N_r(i)$. David (1963) shows that the failure rates $h_1, \dots, h_n$ for all environments compared can be obtained up to proportionality as the solution to the system of equations

$$h_i = \frac{N(i)}{e \sum_{j=1, j \neq i}^{n} [h_i + h_j]^{-1}} , \qquad i=1, \dots, n \tag{14}$$

and Ford (1957) shows that the following iterative solution procedure can be used to solve for the h_i .

$$h_i^{(k+1)} = \frac{N(i)/e}{\sum_{j=1}^{i-1} [h_i^{(k)} + h_j^{(k+1)}]^{-1} + \sum_{j=i+1}^{n} [h_i^{(k)} + h_j^{(k)}]^{-1}} , \qquad i=1, \dots, n \tag{15}$$

where $h_i^{(k)}$ is the kth iteration estimate of h_i (thus we must specify initial estimates) and by convention

$$\sum_{j=1}^{0} [h_1^{(k)} + h_j^{(k+1)}]^{-1} = \sum_{j=n+1}^{n} [h_n^{(k)} + h_j^{(k)}]^{-1} = 0$$

Ford (1957) notes that the estimate obtained is the maximum likelihood estimate and that the solution to (14) is unique and convergence under the conditions that it is not possible to separate the n environments into two sets where all experts deem that no environment in the first set is more severe than any environments in the second set.

Bradley (1957) develops a statistic to test the appropriateness (goodness of fit) of the Bradley-Terry (or NEL) model. Under the null hypothesis that the model is appropriate, the statistic

$$F = 2 \left\{ \sum_{i=1}^{n} \sum_{j=1, j \neq i}^{n} N(i,j) \ln(r(i,j) - \sum_{i=1}^{n} N(i) \ln(h_i) + \sum_{i=1}^{n} \sum_{j=i+1}^{n} e \ln(h_i + h_j) \right\} \tag{16}$$

is (asymptotically) distributed as a chi-square distribution with (n-1)(n-2)/2 degrees of freedom.

4. The Expert Judgment Experiment

Fourteen wiring experts were brought together for a one day workshop in which the expert opinion elicitation took place. Initially, the experts were given an overview of how the wiring environments and the variable breakpoints were determined and how a paired comparison is conducted. Experts were asked to compare the fifteen sample environments given in Table 2. These environments

were selected in consultation with experts not participating in the elicitation. The selection was based on realism, minimal change in environment comparisons, and wide encompassing of the total set of wiring environments.

The experts were asked to fill out 105 (15 taken 2 at a time) survey questions for both the open and shorting failure analysis. Each question compared two environments and the experts were asked to indicate the environment that would produce a failure sooner. The questions were presented in the form depicted in Figure 1 where for ease of comparison, the environments were categorized according to wire, bundle, and zonal properties and the changes from environment 1 to environment 2 were shaded.

4.1 Analysis of the Experts

Experts were analyzed for both individual and group performance. Of the 14 experts, 3 were removed due to failing the statistical test for consistency provided in (7) for both the open and the shorting failure analysis. Responses for an additional 2 experts were removed for failing the statistical test for consistency for the open failure analysis. A comparison of the experts' performance in open and shorting failure analysis is displayed in Figure 2 with dashed lines indicating the critical number of circular triads. It is clear that the experts can be partitioned in to three groups; those that are effective in both open and shorting failure analysis, those that are effective in one analysis but not the other and those that are effective in neither. The remaining group of experts (9 for the open failure analysis and 11

Table 2. Wiring Environments for Paired Comparison

Environment	Wire Guage	Insulation Type	Conductor Type	Splices	Bundle Size	Bundle Protection	Curvature of Bundle	Bundle Orientation.	Ops/Main Traffic	Ops temp/altitude	Vibration	Exp Corrosive Fluid	Exp Conducting Fluid
1	18-22 awg	Hybrid (PI/FP Composite)	Copper	None	Moderate (0.5-1.25 in)	Not Protected (Open)	Low (> 10x)	Horizontal/Vertical Wire	High	Benign (P&T Controlled)	Moderate	No	Yes
2	24-26 awg	Hybrid (PI/FP Composite)	High Streng. Copper Alloy	None	Very Small (< 0.2 in)	Not Protected (Open)	Low (> 10x)	Horizontal/Vertical Wire	High	Benign (P&T Controlled)	Moderate	No	Yes
3	24-26 awg	Hybrid (PI/FP Composite)	Copper	None	Moderate (0.5-1.25 in)	Not Protected (Open)	Low (> 10x)	Horizontal/Vertical Wire	Moderate	Benign (P&T Controlled)	Moderate	No	Yes
4	18-22 awg	Hybrid (PI/FP Composite)	Copper	None	Moderate (0.5-1.25 in)	Some Level of Prot.	Low (> 10x)	Horizontal/Vertical Wire	High	Benign (P&T Controlled)	High	No	Yes
5	18-22 awg	Hybrid (PI/FP Composite)	Copper	None	Large (> 1.25 in)	Not Protected (Open)	Low (> 10x)	Horizontal/Vertical Wire	High	Benign (P&T Controlled)	Moderate	Yes	Yes
6	18-22 awg	Hybrid (PI/FP Composite)	Copper	Non-environmental	Moderate (0.5-1.25 in)	Not Protected (Open)	Low (> 10x)	Horizontal/Vertical Wire	High	Benign (P&T Controlled)	Low	No	Yes
7	18-22 awg	ETFE & other FPs	Copper	None	Moderate (0.5-1.25 in)	Not Protected (Open)	Low (> 10x)	Horizontal/Vertical Wire	High	Benign (P&T Controlled)	Low	No	Yes
8	18-22 awg	Hybrid (PI/FP Composite)	Copper	None	Moderate (0.5-1.25 in)	Not Protected (Open)	High (<= 10x)	Horizontal/Vertical Wire	High	Benign (P&T Controlled)	Moderate	No	No
9	18-22 awg	Hybrid (PI/FP Composite)	Copper	None	Moderate (0.5-1.25 in)	Some Level of Prot.	Low (> 10x)	Horizontal/Vertical Wire	High	D2 (P&T not controlled)	Moderate	No	Yes
10	18-22 awg	Hybrid (PI/FP Composite)	Copper	None	Moderate (0.5-1.25 in)	Not Protected (Open)	High (<= 10x)	Horizontal/Vertical Wire	High	Benign (P&T Controlled)	Low	No	Yes
11	18-22 awg	Hybrid (PI/FP Composite)	Copper	None	Moderate (0.5-1.25 in)	Not Protected (Open)	Low (> 10x)	Longitudinal	High	Benign (P&T Controlled)	Moderate	No	No
12	18-22 awg	Hybrid (PI/FP Composite)	Copper	None	Moderate (0.5-1.25 in)	Not Protected (Open)	Low (> 10x)	Horizontal/Vertical Wire	Low	Benign (P&T Controlled)	High	No	Yes
13	18-22 awg	Polyimide	Copper	None	Moderate (0.5-1.25 in)	Not Protected (Open)	High (<= 10x)	Horizontal/Vertical Wire	High	Benign (P&T Controlled)	Moderate	No	Yes
14	4\0-8 awg	Hybrid (PI/FP Composite)	Aluminum	None	Moderate (0.5-1.25 in)	Not Protected (Open)	Low (> 10x)	Horizontal/Vertical Wire	High	Benign (P&T Controlled)	Moderate	No	No
15	4\0-8 awg	Hybrid (PI/FP Composite)	Copper	None	Moderate (0.5-1.25 in)	Not Protected (Open)	Low (> 10x)	Horizontal/Vertical Wire	High	D2 (P&T not controlled)	Moderate	No	Yes

COMPARISON		
WIRE ENVIRONMENT 1	11	**WIRE ENVIRONMENT 2**

WIRE PROPERTIES			WIRE PROPERTIES	
Wire Gauge	18-22 awg		Wire Gauge	
Conductor Type	Copper		Conductor Type	
Insulation Type	Hybrid (PI/FP Composite)		Insulation Type	
Splices	None		Splices	
BUNDLE PROPERTIES			**BUNDLE PROPERTIES**	
Bundle Size	Moderate (0.5-1.25 in)		Bundle Size	
Bundle Protection	Not Protected (Open)		Bundle Protection	Some Level of Prot.
Curvature of Bundle	Low (> 10x)		Curvature of Bundle	
Bundle Orientation (Shock)	Horizontal/Vertical Wire		Bundle Orientation (Shock)	
ZONAL PROPERTIES			**ZONAL PROPERTIES**	
Ops/Main Traffic	High		Ops/Main Traffic	
Ops Temp/Alt	Benign (P&T Controlled)		Ops Temp/Alt	
Vibration	Moderate		Vibration	High
Exposure to Corrosive Fluid	No		Exposure to Corrosive Fluid	
Exposure to Conductive Fluid	Yes		Exposure to Conductive Fluid	

Figure 1. Paired Comparison Question Format

for the shorting failure analysis) passed the statistical test for group agreement of responses at the 5% level using both (9) and (12). In addition, the goodness-of-fit test provided in (16) was used and it was found that the Bradley-Terry (or NEL) model could not be rejected at the 5% level of significance for either the open failure data or the shorting failure data.
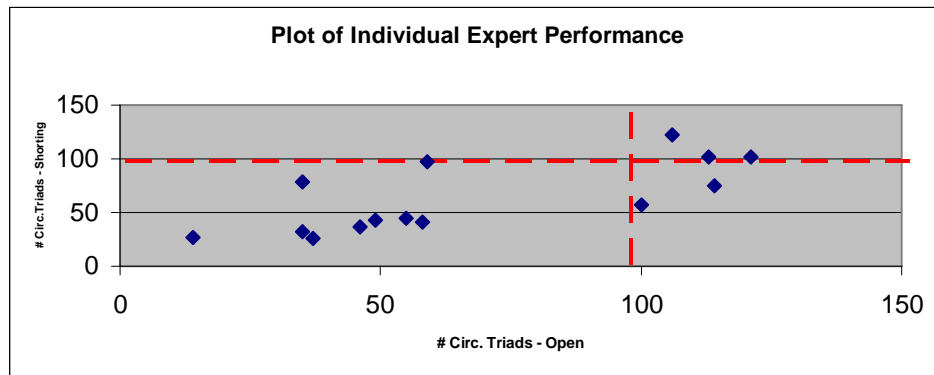

Figure 2 . Caparison of Individual Expert Performance

4.2 Obtaining the Failure Rate Estimates

The PC-based computer program WCOMPAR (available from Delft University of Technology) was used to calculate the solution to (14) using (15) and the estimates (to within a scale constant) of the candidate wiring environment failure rates combined with their joint 90% bounds are provided in Table 3. Note that even within the candidate environments there is a 2 order of magnitude separation in the failure rate estimates.

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Table 3. Bradley-Terry (NEL) Estimates and Joint 90% Confidence Bounds
for the 15 Candidate Wiring Environemnets

Environemnt	Open Failures			Shorting Failures		
	lower	Bradley-Terry Est	upper	lower	Bradley-Terry Est	upper
1	0.016	0.039	0.068	0.020	0.045	0.067
2	0.060	0.121	0.260	0.047	0.085	0.160
3	0.007	0.026	0.047	0.007	0.019	0.039
4	0.017	0.042	0.073	0.031	0.070	0.130
5	0.068	0.119	0.190	0.077	0.150	0.220
6	0.150	0.265	0.420	0.057	0.102	0.170
7	0.004	0.014	0.029	0.006	0.017	0.032
8	0.021	0.050	0.089	0.012	0.028	0.044
9	0.018	0.042	0.063	0.030	0.059	0.110
10	0.019	0.048	0.080	0.019	0.044	0.075
11	0.004	0.020	0.040	0.003	0.012	0.022
12	0.005	0.018	0.041	0.007	0.024	0.038
13	0.110	0.158	0.260	0.160	0.252	0.430
14	0.001	0.008	0.018	0.004	0.012	0.019
15	0.010	0.030	0.055	0.047	0.081	0.120

4.3 Obtaining a Regression Fit

In order to determine the values for covariates X_i, needed for the regression analysis, the experts were also asked to fill out survey questions of the form presented in Figure 5 where for each failure type and each variable, the expert was given a base value and asked by what ratio the environment would become more or less severe as a single variable value was shifted. That is, using Figure 5 for example and considering the variable bundle size. If the current bundle size value is specified as Large (>1.25 in), what factor (1 to 10) of increase (or decrease) in risk of failure would occur if this value is changed to Moderate (0.5-1.25 in). By what factor of increase (or decrease) in risk of failure would occur if this value is changed to Small (0.2-0.5 in) and so forth.

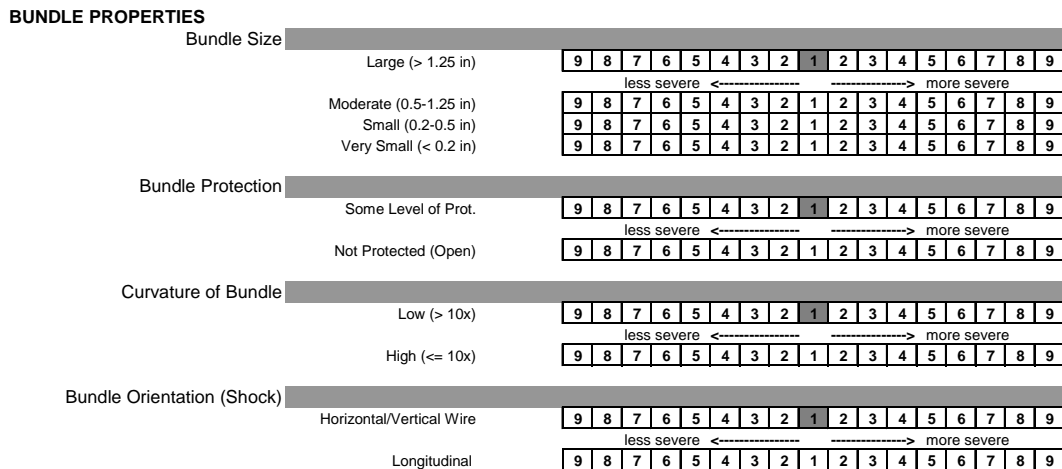**EFFECT OF SINGLE VARIABLES ON OPEN FAILURES**
Page   2

Figure 5. Example Survey for Determining the Values for X_i

On many occasions the experts were in good agreement as depicted in Figure 6 for the variable *bundle protection* when considering open failures. On other occasions, there was considerable

disagreement of the experts as depicted in Figure 7 for the variable *wire gauge* when considering shorting failures. Note that due to the emphasis on ratio values the graphs are presented in log scale.
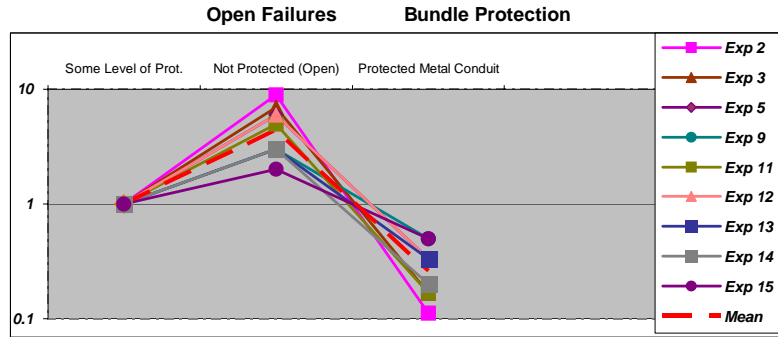

Figure 6. Experts Prediction of Severity Increase as Bundle Protection Moves from Some Level of Protection to Not Protected to Protected Metal Conduit.
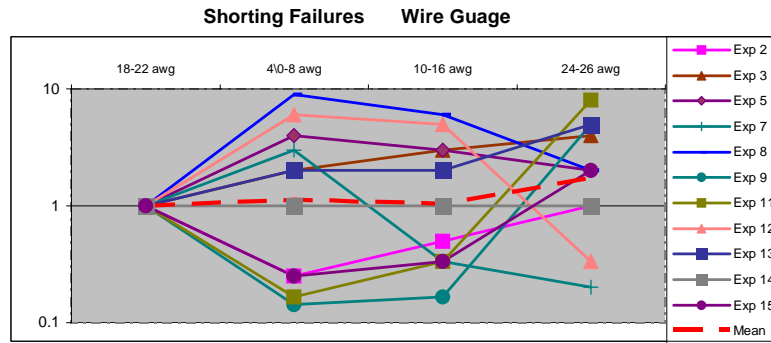

Figure 7. Experts Prediction of Severity Increase as Wire Gauge Moves from 18-22awg to 4/0-8awg to 10-16awg to 24-26aeg.

While these graphs provide a host of additional information, they are not the focus of this paper. By way of clarification of the graph legends, we note that experts were randomly assigned numbers 1 through 15, thus there was no Expert 4. In addition, only the expert scores provided by those experts that passed the consistency test were used in this analysis. Thus as seen from the legends in Figure 6 and 7, experts 1, 6, 7, 8, and 10 were dropped from the open failures values and experts 1, 6 and 10 were dropped from the shorting failures analysis. Note also that the geometric mean of the values is plotted as a dashed line in these figures. The geometric mean for a set of values y_1, …, y_n is given by

$$geom\ mean(y_1,...,y_n) = \prod_{i=1}^{n}(y_n)^{1/n} \tag{17}$$

and is the appropriate measure of central tendency for ratio values. Estimates were made of the magnitude of the increase/decrease in severity of each variable value for both open and shorting failure using the geometric mean. These were used as the coded values for the environmental variables in the regression analysis.

Given the candidate environment failure rate estimates in Table 3 and the coded values for the environmental variables, a backwards elimination selection method selection [see for example Bowerman and O'Connell (2000). page 530] was used to determine the most appropriate model relating the expert responses to the coded environmental variable values for both open and shorting failures.

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These regression results presented in Figure 8 and 9. Variables that do not appear in

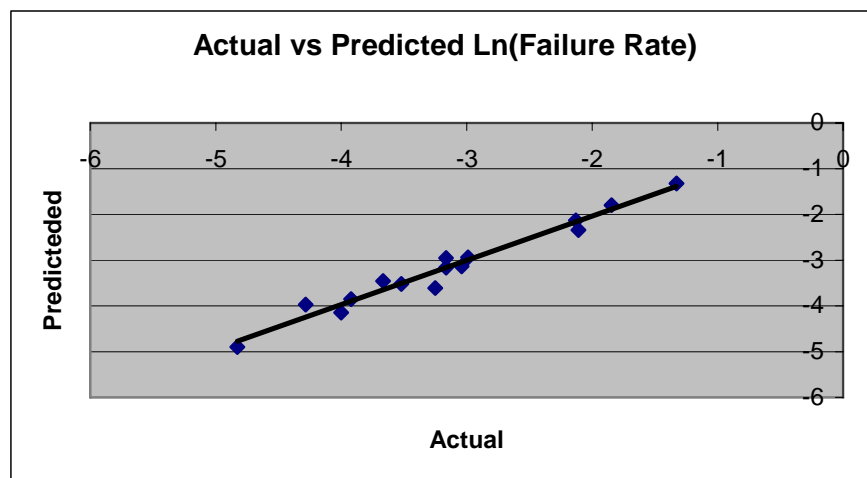| SUMMARY OUTPUT OPEN FAILURE ANALYSIS						
Regression Statistics						
Multiple R	0.9987					
R Square	0.9975					
Adjusted R Square	0.7929					
Standard Error	0.2868					
Observations	15					
ANOVA						
	df	*SS*	*MS*	*F*	*Significance F*	
Regression	10	161.4031	16.1403	196.2824	0.0001	
Residual	5	0.4112	0.0822			
Total	15	161.8142				
	Coefficients	*Standard Error*	*t Stat*	*P-value*		
Intercept	0	#N/A	#N/A	#N/A		
Wire Guage	0.4535	0.1343	3.3770	0.0197		
Insulation Type	2.0738	0.6439	3.2209	0.0234		
Conductor Type	-0.4380	0.1701	-2.5745	0.0498		
Splices	0.5639	0.0781	7.2246	0.0008		
Curvature of Bundle	0.5013	0.2000	2.5061	0.0541		
Bundle Orientation	-8.1221	0.9121	-8.9051	0.0003		
Ops/Main Traffic	0.2014	0.0560	3.5950	0.0156		
Ops temp\altitude	0.2050	0.1236	1.6585	0.1581		
Vibration	0.2239	0.0924	2.4218	0.0600		
Exp Corrosive Fluid	0.4742	0.1026	4.6237	0.0057		

Figure 8. Regression Output for Open Failures

SUMMARY OUTPUT SHORTING FAILURE ANALYSIS

Regression Statistics	
Multiple R	0.9923
R Square	0.9846
Adjusted R Square	0.9462
Standard Error	0.2143
Observations	15

ANOVA

	df	SS	MS	F	Significance F
Regression	10	11.7579	1.1758	25.6012	0.0034
Residual	4	0.1837	0.0459		
Total	14	11.9416			

|	Coefficients	Standard Error	t Stat	P-value
Intercept	-13.0056	1.0868	-11.9666	0.0003
Wire Guage	0.9203	0.2697	3.4119	0.0270
Insulation Type	1.7154	0.4447	3.8577	0.0182
Splices	1.1536	0.1902	6.0654	0.0037
Bundle Protection	0.2512	0.1276	1.9692	0.1203
Curvature of Bundle	0.3723	0.0880	4.2288	0.0134
Ops/Main Traffic	0.4368	0.0717	6.0928	0.0037
Ops temp\altitude	0.5998	0.1470	4.0796	0.0151
Vibration	0.6605	0.1202	5.4976	0.0053
Exp Corrosive Fluid	0.3456	0.0613	5.6373	0.0049
Exp Conducting Fluid	0.2873	0.0419	6.8593	0.0024
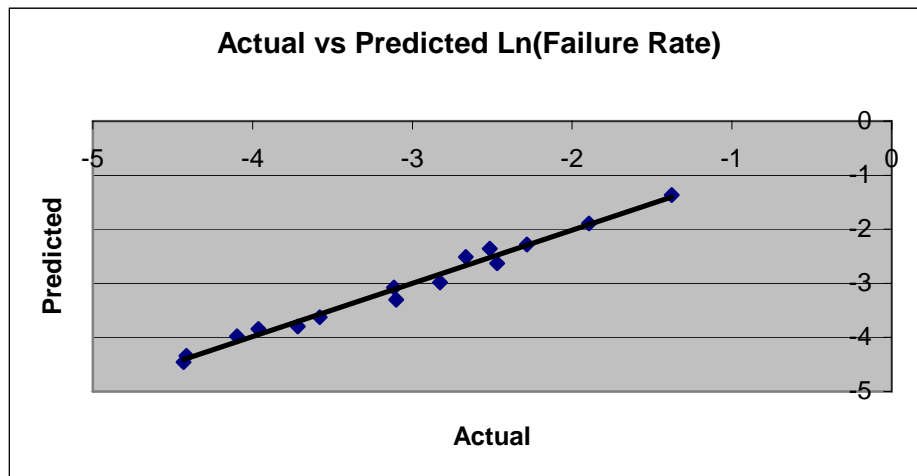
Figure 9. Regression Output for Shorting Failures

the figures were deemed not significant in their contribution to the regression in explaining the Ln(failure rate) variation as a function of the environment and are thus assigned a coefficient value of 0. Variables were dropped whose p-value was significantly above 0.20 during the backward elimination process. While this is fairly lenient, emphasis was placed on including as many variables as possible and

within reason. The unusually high multiple R square value is to be expected due to the small number of degrees of freedom. However, the graphical fit appears to be more then reasonable.

4.4 Rescaling of the Failure Rate Surface

One of the environments used in the paired comparison study (environment 10) is characteristic of the environment for emergency path lighting. Failure data is available for this environment as reported failures are mandated by the Federal Aviation Administration (FAA). In addition, the Air Operators Utilization Reports found in the FAA website

http://www.faa.gov/data_statistics/aviation_data_statistics/

provides the exposure time component for equation (5). Calculation of this estimate still requires some data cleaning and has not been completed at this time.

5. Conclusions

We have illustrated a potentially useful procedure for estimating wire failure probabilities. As with any procedure there are also potential drawbacks. The procedure can yield estimates for any failure environment and can therefore be useful for a more thorough risk analysis. This can be of help in both the initial design and the retrofit design of the EWIS of an aircraft. Currently a PC based EWIS risk assessment tool is being developed by Lectromechanical Design Company for such purposes. The failure rate surfaces estimated here will be inputs for this tool.

While it is always advisable to use actual failure data when it is available, it is simply to scarce in this application. Though the use of expert judgment is not new, it may be new in this arena and it may take time for the industry to accept its use. Great care should be taken to not accept these numbers as given truths but rather as estimates based on initial modeling and data analysis. Confidence intervals for these estimates can be obtained using simulation and could prove to be quite wide.

Comparison of these failure rate estimates to estimates obtained from actual data for various environments would be the obvious next step in proofing the procedure. In addition, investigation of dependence of environmental variables should be undertaken using expanded expert elicitation.

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